15-344 Combinatorics on Oct 13, hours 13-14: The 4CT, chromatic polynomials

Thursday, September 17, 2015 7:59 PM

Read Along: Sects 2.3, 2.4, 3.1.

TA Office hours: Tuesdays 6-7PM at 215 Huron, room 1012, starting today.

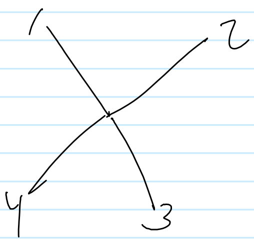
WARNING: I may be telling lies today!

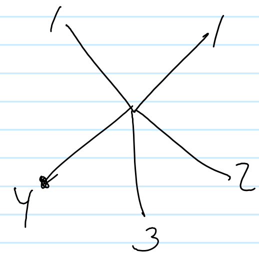
Thm (The 5CT) every planar graph can be 5-coloured.

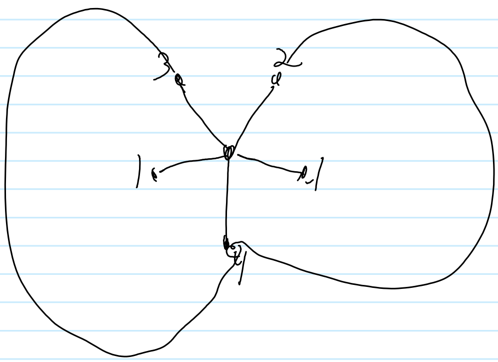
Lemma A planar graph has at least one vertex of valency ≤ 5. on board

---

pf of the 4CT. Cases:







First mentioned Mobius 1840.
Conjectured Guthrie 1852.
Mis-proven Kempe 1879.
Bug found Heawood 1890.
Proven Appel-Haken 1976.
Coq proof Gonthier 2005.

left out

No lies from here on.

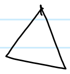
Aside: Edge colouring

Thm 4CT ⟺ Every planar trivalent graph has an edge 3-colouring.

PM: This may have been too hard and not interesting enough, without prior knowledge of group theory.

The chromatic polynomial $P_k(G)$

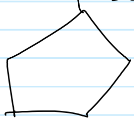
Examples 0. Empty(n)

1.  K_n

2. L_n:  n vertices.

done line

3. Trees. (connected, no circuits)

4. C_5:  } no do

Then $G_{+(x,y)}$ $G_{x=y}$ $G_{-(x,y)}$

Thm

1. If $(x,y) \notin E$, $P_k(G) = P_k(G_{+(x,y)}) + P_k(G_{x=y})$.
2. If $(x,y) \in E$, $P_k(G) = P_k(G_{-(x,y)}) - P_k(G_{x=y})$.

Do example 4.

Little on trees:

Thm For a connected graph T, TFAE:

1. T has no circuits.
2. Let a be some vertex of T. For any other vertex x of T there is a unique path from a to x.
3. There is a unique path between any two $x \neq y \in V$
4. T is "minimally connected": it becomes

disconnected upon the removal of any edge in T.

<u>Pf</u> $1 \Rightarrow 4 \Rightarrow 3 \Rightarrow 2 \Rightarrow 1$.

<u>Def</u> Such T is a tree.

<u>Thm</u> In a tree, $e = v - 1$.