15-344 Combinatorics on Nov 3, hours 22-23: Basic Enumeration

Tuesday, November 3, 2015
Rend Along: 5.1 5.2
Exam discussion \& return @ 4:45.
Go over questions 1-8:

$$
\left.\begin{array}{l}
2 \text { min: quit discussion } \\
\text { ins: convince you fried. } \\
\text { in class discussion, }
\end{array}\right\} \begin{aligned}
& \text { each } \\
& \text { question. }
\end{aligned}
$$

Extra Q: How many way are Non to distribute


Results
160 students took the exam, and 14 asked for medical exemptions (a record). The full list of results, before appeals, was (median underlined):

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100100100100100100999998989898989898989898989898989797979797979796969696969696
9696969595959595959594949393939393929291919191919089898989898888888888878787
8787878686\underline{8585 85 83 83 83 83 82 82 828181818181808080 807979787877777675757574737372}
7271717069686767656563636362626261606059595757565451515048484644424240393834
3434242322201814105
```

The exams will be returned in class on Tuesday November 3.
Appeals.
Remember! We try hard yet grading is a difficult process and mistakes always happen - solutions get misread, parts are forgotten, grades are not added up correctly. You must read your exam and make sure that you understand how it was graded. If you disagree with anything, don't hesitate to complain! (Though first consider very carefully the possibility that the mistake is actually yours). Your first stop should be the person who graded the problem in question, and only if you cant agree with him you should appeal to Dior.
Dror graded problem number 4 and did the data entry. Gaurav Patil graded everything else.
The deadline to start the appeal process is Thursday November 12 at 1PM. Once you've started the process by talking to Dror or to Gaurav, it ends when a final decision is made, with no deadline.

From <http://drorbn.net/index.php?title=15-344/Term Test>

$$
\begin{aligned}
& \text { Less than 80: do better } \\
& \text { 50: you should be deeply concernedly }
\end{aligned}
$$

## Example 1: Rolling Dice

Two dice are rolled, one green and one red. Each die has faces numbered 1 through 6.
(a) How many different outcomes of this procedure are there?
(b) What is the probability that there are no doubles (not the same value on both dice)?
Example 2: Arranging Books
There are five different Spanish books, six different French books, and eight different Transylvanian books. How many ways are there to pick an (unordered) pair of two books not both in the same language?

## Example 3: Sequences of Letters

How many ways are there to form a three-letter sequence using the letters $a, b$, $c, d, e, f$ (a) with repetition of letters allowed? (b) without repetition of any letter? (c) without repetition and containing the letter $e$ ? (d) with repetition and containing $e$ ?

Example 4: Nonempty Collections
How many nonempty different collections can be formed from five (identical) apples and eight (identical) oranges?
Example 1: Ranking Wizards
How many ways are there to rank $n$ candidates for the job of chief wizard? If the ranking is made at random (each ranking is equally likely), what is the probability that the fifth candidate, Gandalf, is in second place?
Example 2: Arrangements with Repeated Letters
How many ways are there to arrange the seven letters in the word SYSTEMS? In how many of these arrangements do the three Ss appear consecutively?

## Example 3: Binary Sequences

How many different 8-digit binary sequences are there with six is and two Os? Example 4: Poker Probabilities
(a) How many 5-card hands (subsets) can be formed from a standard 52-card deck?
(b) If a 5-card hand is chosen at random, what is the probability of obtaining a flush (all five cards in the hand are in the same suit)?
(c) What is the probability of obtaining three, but not four, Aces?

Example 5: Forming Committees
A committee of $k$ people is to be chosen from a set of seven women and four men. How many ways are there to form the committee if
(a) The committee consists of three women and two men?
(b) The committee can be any positive size but must have equal numbers of women and men?
(c) The committee has four people and one of them must be Mr. Begins?
(d) The committee has four people and at least two are women?
(e) The committee has four people, two of each sex, and Mr. and Mrs. Baggins cannot both be on the committee?
Example 2: (continued) Arrangements with Repetitions
How many arrangements of the seven letters in the word SYSTEMS have the E occurring somewhere before the M ? How many arrangements have the E somewhere before the M and the three Ss grouped consecutively?

## Example 6: Counting Defective Products

A manufacturing plant produces ovens. At the last stage, an inspector marks the ovens $A$ (acceptable) or $U$ (unacceptable). How many different sequences of 15 As and $U \mathrm{~s}$ are possible in which the third $U$ appears as the twelfth letter in the sequence?

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Example 7: Probability of Repeated Digits
What is the probability that a 4-digit campus telephone number has one or more repeated digits?
skipped for lack of interest.


