15-344 Combinatorics on Dec 8, hours 35-36: More Fibonacci numbers

Monday, November 23, 2015 7:11 AM

Evaluations responses: 43/167 https://course-evals.utoronto.ca/blue/a.aspx?l=591_1709_AAAAAAABgaM

HW 10 details.

Final exam schedule & details.

Agenda: Fibonacci: def, gen-fun, formula, identities.

Read Along: Your notes, then 7.3, 7.5

Fibonacci. How many ways to climb to the top of a staircase on n stairs, climbing one or two stairs each time?

Sol'n $F_0=1$ $F_1=1$ $F_2=2$, $F_n=F_{n-1}+F_{n-2}$

on board

Generating fnctn:

Method 1: $f=\sum_{k=0}^{\infty}(x+x^2)^k=\frac{1}{1-x-x^2}$

Method 2: $(1-x-x^2)f=1$

Finding F_n: // instead find G_n, where

$G_0=1$ $G_1=4$ $G_n=5G_{n-1}-6G_{n-2}$

$G_2=14$ $(2-x)(3-x)=6-5x+x^2$

Guess $G_n=\alpha^n$ ...... get $\Rightarrow G_n=2\cdot 3^n-2^n$

With Fibonacci, get

Pensieve header: A formula for the Fibonacci numbers, plus some tidbits.

In[1]:= `{λ1, λ2} = x /. Solve[x^2 - x - 1 == 0]`

Out[1]= $\left\{\frac{1}{2}\left(1-\sqrt{5}\right), \frac{1}{2}\left(1+\sqrt{5}\right)\right\}$

In[2]:= `F[n_] := Expand[a λ1^n + b λ2^n];`
`Solve[F[0] == 1 ∧ F[1] == 1, {a, b}] /. Rule → Set`

Out[3]= $\left\{\left\{\frac{1}{10}\left(5-\sqrt{5}\right), \frac{1}{10}\left(5+\sqrt{5}\right)\right\}\right\}$

In[4]:= `F[n]`

Out[4]= $2^{-1-n}\left(1-\sqrt{5}\right)^n - \frac{2^{-1-n}\left(1-\sqrt{5}\right)^n}{\sqrt{5}} + 2^{-1-n}\left(1+\sqrt{5}\right)^n + \frac{2^{-1-n}\left(1+\sqrt{5}\right)^n}{\sqrt{5}}$

In[5]:= `Table[F[n], {n, 0, 10}]`

Out[5]= {1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89}

In[12]:= `{F[21] / F[20], GoldenRatio} // N`

Out[12]= {1.61803, 1.61803}

In[6]:= `n = 30; F[n]`

Out[6]= 1 346 269

Homework (not to be submitted). What is the story told by each of the following formulas?

In[7]:= $\left\{F\left[\frac{n}{2}\right]^2 + F\left[\frac{n}{2}-1\right]^2, \sum_{k=0}^{n/2}\text{Binomial}[n-k, k], \sum_{k=0}^{n-2}F[k], \sum_{k=0}^{n/2-1}F[n-2k-1]\right\}$

Out[7]= {1 346 269, 1 346 269, 1 346 268, 1 346 268}

In[8]:= `Table[Binomial[n, k], {n, 0, 10}, {k, 0, 10}] // MatrixForm`

Out[8]//MatrixForm=

$$\begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 3 & 3 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 4 & 6 & 4 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 5 & 10 & 10 & 5 & 1 & 0 & 0 & 0 & 0 & 0 \\
1 & 6 & 15 & 20 & 15 & 6 & 1 & 0 & 0 & 0 & 0 \\
1 & 7 & 21 & 35 & 35 & 21 & 7 & 1 & 0 & 0 & 0 \\
1 & 8 & 28 & 56 & 70 & 56 & 28 & 8 & 1 & 0 & 0 \\
1 & 9 & 36 & 84 & 126 & 126 & 84 & 36 & 9 & 1 & 0 \\
1 & 10 & 45 & 120 & 210 & 252 & 210 & 120 & 45 & 10 & 1
\end{pmatrix}$$