Many people have asked me about the question about colouring the regions made by the n-circles. While some of you did provide complete solutions, this page should help you understand what happens behind the scenes in the proof.(will help you understand when this property will hold).

First off you must see the picture as a graph which requires region coloring. So vertices will be intersections of circles. i.e. a point $p$ is a vertex if atleast 2 of the circles pass thorough it. We consider the arcs as edges. Thus we have our rull of the mill planar graph whose regions we have to color. Now look at any vertex $P$. Every circle that passes through this point creates 2 edges at the point. So $degree(P)$ must be twice the number of circles passing through and so must be even.

Now remember the graph you made to convert a region coloring problem to vertex coloring problem? every vertex becomes a region and every region a vertex? edges remain as edges? This new graph will be such that every region will have even degree.

Now for any non self intersecting cycle in this new graph we have that the length of cycle is : the sum of all the degrees of regions inside it minus twice the total number of edges inside the cycle chosen. Since each degree of the region is even, we see that the length of the cycle itself is even.

now even self intersecting cycles will have even lengths as their length will be equal to sum of length of non-selfintersecting cycles.

So we have length of every cycle is even which tells us that the graph is bipartite. which is same as two vertex colourable.

Bullet points.

degree of each vertes in the region coloring graph is even.
convert the above property to the corresponding vertex coloring graph
the property becomes each region is even cycle.
realize every cycle is a sort of combination of these regions making each cycle even
making graph bipartite