
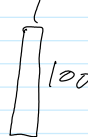


course evals: 18/112.

HW9 on web by midnight.

Riddle. Can you cover  with $99 \times$ 

Determinants:

Mention the name "Gaussian Elimination".

1. Applications: mention 2, prove 1, use none
2. Formulas: Discuss just one.
3. Basic properties: our core subject.

det is a certain specific function, $\det: M_{n \times n}(F) \rightarrow F$, which we will properly define later; $\det(A) \equiv |A|$.

$$1. A \text{ invertible} \iff \det(A) \neq 0$$

$$2. \left| \det \begin{pmatrix} -r_1 \\ -r_2 \\ \vdots \\ -r_n \end{pmatrix} \right| = \text{vol} \begin{pmatrix} \text{parallelepiped generated} \\ \text{by } r_1 \dots r_n \end{pmatrix}$$

$$|(a_{11})| := a_{11} \quad \left| \begin{pmatrix} a_{11} & \dots & a_{1n} \\ \vdots & & \vdots \\ a_{n1} & \dots & a_{nn} \end{pmatrix} \right| := \sum_{j=1}^n (-1)^{1+j} a_{1j} |A_{1j}^{11}|$$

Examples $\begin{vmatrix} a & b \\ c & d \end{vmatrix}$ $\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix}$ "Cofactor expansion"

Basic properties: 0. $\det(I) = 1$.

Proof now!

$$1. \det(E_{ij}^1 A) = -\det(A) \quad [\det E_{ij}^1 = -1]$$

"Exchanging two rows flips the sign of det"

$$2. \det(E_{ic}^2 A) = c \det A \quad [\det E_{ic}^2 = c]$$

"multiplying a row by c multiplies det by c "
even for $c=0$

$$3. \det(E_{iij,c}^3 A) = \det A \quad [\det E_{iij,c}^3 = 1]$$

"adding c times one row to another does not change det"

Proofs later.

Thm Using these properties, the determinant of

any $n \times n$ matrix A can be computed.

done line

PF Row reduce A keeping track of the affect on $\det A$;

For v.v.e.f B , $\det B = 1$ if $B = \begin{pmatrix} 1 & & \\ & \ddots & \\ & & 1 \end{pmatrix}$, and

$\det B = 0$ if B has a row of 0's.

Examples

$$\det \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}, \quad \det \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$$

old done line

Corollary All that there is to know about determinants can be deduced from 0-3; also if \det' satisfies 0-3, then $\det' = \det$.

Thm A is invertible iff $\det(A) \neq 0$

Thm If $A = E_1 \dots E_n$ is a product of elementary matrices, then $\det A = \det(E_1) \cdot \det(E_2) \dots \det(E_n)$

Claim For square matrices, AB invertible $\Leftrightarrow A$ & B are inv.

$$\Leftarrow (AB)^{-1} = B^{-1}A^{-1}$$

$\Rightarrow B(AB)^{-1}$ is a right inverse for A , & for square matrices, if $AC = I$ then also $CA = I$.

Thm $\det A \cdot B = \det A \cdot \det B$

Thm $\det A^T = \det A$

Thm Everything that's true for rows is also true for columns.

Skipped extras: 1. Other formulas for \det . (row/col expansions, permutations)

2. A \det formula for A^{-1} & Kramp's law. 1. It's in all the books.

2. I've never used it in my life.

... recall the formula for \det s & sketch the proof of the basic properties.

$$|(a_{11})| := a_{11} \quad \left| \begin{pmatrix} a_{11} & \dots & a_{1n} \\ \vdots & & \vdots \\ a_{n1} & \dots & a_{nn} \end{pmatrix} \right| := \sum_{j=1}^n (-1)^{1+j} a_{1j} |A_{1j}^{\wedge}|$$

Then prove:

1. Linear in the first row.
2. Multilinear in the rows.
3. Vanishes if the first two rows are equal.
4. Vanishes if two adjacent rows are equal.
5. Switches sign if two adjacent rows are interchanged.
6. Switches sign whenever two rows are interchanged.
7. E_{ij}^2 & $E_{ij,c}^2$ behaviour.