240AlgebraI-141020 Hour 18: Lagrange Interpolation

September-18-14 6:29 PM

Read Along 1.6 – 1.7.

HW5 on web by midnight!

Term test tomorrow 1:10 – 3 PM, HS 610.

My expectations: 1. complete Mastery of the material.

2. Yes, meaning every single proof.

Find old TT's on previous year's web sites

TA office hours – Today 12–3 Nikita near BA6178

4–5 Boris near BA6178

Tue 10–12 Boris at SS1071

Today: Finish bases, Lagrange interpolation.

Reminder.

3. If V is finite-dimensional and $W \subset V$ is a subspace, then W is f.d. and $\dim W \leq \dim V$.

Pf Let β be a linearly indep sequence in W which is of maximal size. [Such exists, as argued before]

It is a generating set [also as argued before]

If also $\dim W = \dim V$, then $W = V$.

If also $\dim W < \dim V$, then any basis of W can be extended to a basis of V. (use replacement thm)

Fishy Thm: Every v.s. has a basis. [Including $\mathbb{R}/\mathbb{Q}$]

The Lagrange interpolation formula:

Let x_i be distinct pts in $\mathbb{R}/F$

$i = 1, \dots, n+1$

Let y_i be any pts in $\mathbb{R}/F$.

Q Can you find a polynomial $P \in P_n(\mathbb{R})$ s.t. $P(x_i) = y_i$?

Is it unique?

Who cares?

* Scientists.
* Computer drawing programs.

Follow through w/ example.

$p(0) = 5$

$p(1) = 2$

$p(2) = ?$

$P_1 = \frac{(x-1)(x-3)}{3} = \frac{1}{3}(x^2 - 4x + 3)$

$P_2 = \frac{x(x-3)}{-2} =$

* Computer drawing programs.

$p(1) = 2$
$p(3) = 2$

$P_2 = \frac{x(x-3)}{-2} =$
$P_3 = \frac{x(x-1)}{6} = \ldots$

$p = x^2 - 4x + 5$

Solution Let $\tilde{P}_i(x) = \prod_{j \neq i} (x - x_j)$

Then $\tilde{P}_i(x_j) = \begin{cases} 0 & j \neq i \\ \neq 0 & i = j \end{cases}$

done live

Set $P_i(x) = \tilde{P}_i(x) / \tilde{P}_i(x_i) = \ldots$

Then * $P(x) := \sum y_i P_i(x)$ satisfies $P(x_i) = y_i$

* $\beta = \{P_1 \ldots P_{n+1}\}$ is lin. indep.
* $\Rightarrow \beta$ is a basis
* Every $f \in P_n(\mathbb{R})$ can be expressed as a lin. comb. of the P_i in a unique way.
* If $q(x)$ also satisfies $q(x_i) = y_i$, then $q(x) = P(x)$.
* Therefore the solution to our problem is unique
* Aside: If $\forall i$, $P(x_i) = 0$, then $P = 0$

(so a non-zero polynomial of degree n has at most n roots.)