240Algebral-140924 Hours 8-9: Vector spaces, subspaces

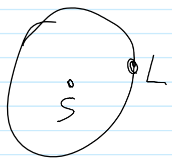
September-18-14 6:29 PM

Class photo today?

Office Hours. Wed 3-4 this week & next.

Read Along Sections 1.1–1.4 of textbook.

Riddle Along



$V_L = 4V_S$

Today: Vector spaces, subspaces

Reminder: A v.s. over a field F is a set V, with a special element $0_V \in V$, a binary $+ : V \times V \to V$ and a binary $\cdot : F \times V \to V$, s.t.

VS1. $x+y=y+x$ VS2: Assoc.

start line

VS3. 0 VS4: $-$

VS5: $1 \cdot x = x$ VS6 $a(bx) = (ab)x$

VS7 $a(x+y)$ VS8 $(a+b)x$

Examples: 1. F^n

2. $M_{m \times n}(F)$

3. $\mathcal{F}(S,F)$ S a set; Bytes / bits

4. Polynomials

done line

5. $\mathbb{C}/\mathbb{R}$ $\mathbb{R}/\mathbb{Q}$ "Galois theory"

Thm 1. Cancellation law: additive, $2\times$ multiplicative.

2. 0_V is unique

3. negatives are unique.

5. $0 \cdot x = 0$ 6. $a \cdot 0 = 0$

7 $(-a)x = -(ax) = a(-x)$

8. $cv = 0 \iff c = 0 \vee v = 0$

Def $W \subset V$ is a "subspace" if it is a vector space with the operations it inherits from V.

Thm $W \subset V$ is a subspace iff it is non-empty & "closed under addition and under multiplication by a scalar"

Examples 1. $\{A \in M_{n \times n}(F) : A^t = A\}$

2. $\{A \in M_{n \times n}(F) : \operatorname{tr} A = 0\}$

3. If W_1 & W_2 are subspace of V, The so is $W_1 \cap W_2$ (What about unions?)

Goal: Every v.s. has a "basis". So while we don't have to use coordinates, we can.

Def u is a l.c. of $u_1 \ldots u_n$ if $\exists a_i \in F$ s.t. $u = \sum a_i u_i$

Examples 1. Vitamins as in the handout

2. In $P_3(\mathbb{R})$, $2x^3 - 2x^2 + 12x - 6$ is a l.c. of $x^3 - 2x^2 - 5x - 3$ and $3x^3 - 5x^2 - 4x - 9$

but $3x^3 - 2x^2 + 7x + 8$ isn't.

Thm If $\{u_i\} \subset V$ then $W = \operatorname{span}(u_i) := \{\text{all l.c. of the } u_i\text{'s}\}$ is a subspace of V.