

class photo today!

Office Hours. Wed 3-4 This week & next.

Read Along Sections 1.1-1.4 of textbook.

Riddle Along

$$\begin{array}{c} \circ \\ | \\ \text{S} \end{array} \circ \text{L} \quad V_L = 4V_S$$

Today: vector spaces, subspaces

Reminder. A v.s. over a field F is a set V , with a special element $0 \in V$, a binary $+$: $V \times V \rightarrow V$ and a binary \cdot : $F \times V \rightarrow V$, s.t.

VS1. $x + y = y + x$

VS2: Assoc.

start

VS3. 0

VS4: $-$

line

VS5: $1 \cdot x = x$

VS6 $a(bx) = (ab)x$

VS7 $a(x+y)$

VS8 $(a+b)x$

Examples: 1. F^n

2. $M_{m \times n}(F)$

3. $\mathcal{F}(S, F)$ S a set; Bytes / bits

4. Polynomials

5. \mathbb{C}/\mathbb{R}

\mathbb{R}/\mathbb{Q}

"Galois theory"

done

line

Thm 1. Cancellation law: additive, $2 \times$ multiplicative.

2. 0_V is unique

3. negatives are unique.

5. $0 \cdot x = 0$

6. $a \cdot 0 = 0$

7. $(-a)x = -(ax) = a(-x)$

$$\S. CV = 0 \Leftrightarrow C = 0 \vee V = 0$$

Def $W \subset V$ is a "subspace" if it is a vector space with the operations it inherits from V .

Thm $W \subset V$ is a subspace iff it is non-empty & "closed" under addition and under multiplication by a scalar"

Examples 1. $\{A \in M_{n \times n}(F) : A^t = A\}$

2. $\{A \in M_{n \times n}(F) : \text{tr } A = 0\}$

3. If W_1 & W_2 are subspace of V ,
Then so is $W_1 \cap W_2$ (What about unions?)

Goal: Every V is. has a "basis". So while we don't have to use coordinates, we can.

Def u is a l.c. of u_1, \dots, u_n if $\exists \alpha_i \in F$
s.t. $u = \sum \alpha_i u_i$

Examples 1. Vitamins as in the handout

2. In $P_3(\mathbb{R})$, $2x^3 - 2x^2 + 12x - 6$ is

a l.c. of $x^3 - 2x^2 - 5x - 3$

and $3x^3 - 5x^2 - 4x - 9$

but $3x^3 - 2x^2 + 7x + 8$ isn't.

Thm $\forall \{u_i\} \subset V$ then $W = \text{span}(u_i) := \{ \text{all l.c. of the } u_i \}$

is a subspace of V .

TABLE 1.1 Vitamin Content of 100 Grams of Cereals (Data)

Cereal	Thiamin	Riboflavin	Niacin	Iron
Wheat	0.0001	0.0001	0.0001	0.0001
Barley	0.0001	0.0001	0.0001	0.0001
Oats	0.0001	0.0001	0.0001	0.0001
Rye	0.0001	0.0001	0.0001	0.0001
Millet	0.0001	0.0001	0.0001	0.0001
Buckwheat	0.0001	0.0001	0.0001	0.0001
Sorghum	0.0001	0.0001	0.0001	0.0001
Maize	0.0001	0.0001	0.0001	0.0001
Rice	0.0001	0.0001	0.0001	0.0001
Wheat (enriched)	0.0001	0.0001	0.0001	0.0001

Linear combinations of polynomials:

$$\begin{aligned}
 &A = (2, -2, 12, -6) \\
 &B = (1, -2, -5, -3) \\
 &C = (3, -5, -4, -9) \\
 &D = (3, -2, 7, 8)
 \end{aligned}$$