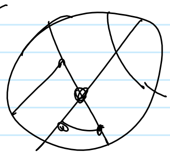
Wednesday-7 AKT on 140226: Graph cohomology and the construction of a UFTI

February-25-14 11:11 AM

$\Gamma = \{\text{all embeddings } \gamma: S^1 \to \mathbb{R}^3\}$

First Goal: a combinatorial model to (a part of) $\Omega^*(\Gamma)$ } on board

$\bar{\mathcal{D}} = \Big\langle$ 
- 3-valent and up vertices
- directed internal edges/all
- ordered internal vertices/all
- no bubbles

$\Big\rangle \Big/ \text{signs}$

For $D \in \bar{\mathcal{D}}$, $I(D) \in \Omega^{m(D)}(\Gamma)$ is

$$I(D) = \pi_* \Phi_D^* \omega^{E_i(D)}$$

$$\Phi_D : C_D^\gamma \longrightarrow (S^2)^{E_i(D)}$$

$$\begin{array}{ccc} C_D^\gamma & \longrightarrow & T \\ & & \downarrow \pi \\ & & \Gamma \end{array}$$

$$m(D) = \text{"the form deg of } D\text{"} = 2|E_i(D)| - |V_s(D)| - 3|V_i(D)|$$

$$= 2|E(D)| - 3|V(D)| = \sum_{v \in V(D)} (\deg v) - 3$$

$$n(D) = -\chi(D) = |E| - |V| = \text{"the type of } D\text{"}$$

$$dI(D) = (\partial\pi)_* \Phi_D^* \omega^{E_i(D)}$$

$$\partial C_D^\gamma = \underbrace{\bigcup_{\substack{\text{edges of}\\ D \text{ except chords}}} (S^2 \text{ or } I(t_b)) \times C^\gamma_{D/e}}_{\text{principal faces}} \;\cup\; \underbrace{\bigcup_{\text{bigger subdiagrams}} \text{mess}}_{\substack{\text{hidden faces}\\ \text{(wishful thinking)}}}$$

Wishful Thinking: The contribution to $dI(D)$ from the hidden faces vanishes.

done line — I should have stated the thm before its derivation.

Corollary: (mod WT) The following diagram commutes:

$$\begin{array}{ccccccc} \bar{\mathcal{D}}_n^\bullet : & \longrightarrow & \bar{\mathcal{D}}_n^m & \xrightarrow{d} & \bar{\mathcal{D}}_n^{m+1} & \longrightarrow \\ \downarrow I & & \downarrow I & & \downarrow I & \\ \Omega^\bullet : & \longrightarrow & \Omega^m(\Gamma) & \xrightarrow{d} & \Omega^{m+1}(\Gamma) & \longrightarrow \end{array}$$

where $dD = \sum_{e \in E(D)} D/e$ $\qquad [d^2 = 0!]$

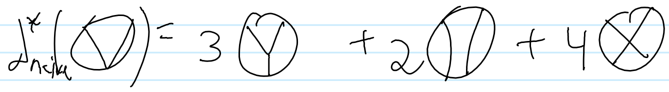
given $v_0 \xrightarrow{e} v_1 \longrightarrow$ ⋮

1. remove v_0
2. remove v_1
3. insert v.

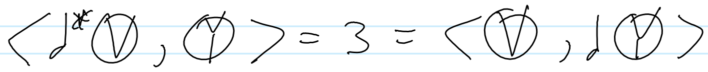
In particular, we have $I_n : H^0(\bar{\mathcal{D}}_n) \longrightarrow \mathcal{V}^0(\Gamma)$
$= \{\text{knot invariants}\}$

set $Z_0 = \sum_n I_n^* : \{\text{knots}\} \longrightarrow \widehat{\bigoplus_{n \geq 0}} [H^0(\bar{\mathcal{D}}_n)]^*$

$$(H^0(\mathcal{D}))^* = (\ker d)^* = (\bar{\mathcal{D}}^0)^* / \operatorname{im} d^* : (\bar{\mathcal{D}}^1)^* \longrightarrow (\bar{\mathcal{D}}^0)^*$$
$$= \mathcal{D}_0 / \operatorname{im} d^* : \bar{\mathcal{D}}_1 \to \bar{\mathcal{D}}_0$$



$\mathcal{D}_m = (\bar{\mathcal{D}}^m)^* = \bar{\mathcal{D}}^m$; yet set $\langle \bar{D}, \underline{D}' \rangle = |\mathrm{Aut}(D)| \delta_{DD'}$



In general $d^* D$ = sum over all ways of breaking a vertex in D.

<u>Prop</u> $H^0(\mathcal{D}^\bullet)^* \cong \mathcal{A} = \mathcal{D} / \begin{matrix} STU \\ IHX \end{matrix}$

<u>prop</u>

$$Z_0(\gamma) = \sum_{D \in \mathcal{D}} \frac{1}{|\mathrm{Aut}(D)|} I(D)(\gamma) = \sum_{D \in \mathcal{D}} \frac{1}{|\mathrm{Aut} D|} \int_{C_D^\gamma} \Phi_D^* \, w^{E_i(D)}$$

<u>prop</u> If invariant, Z_0 is a UFTI.