Wednesday-5 AKT on 140205: The Fulton-MacPherson compactification, 1

February-05-14 8:34 AM

* "Compactification" handout

Blatantly False theorem.

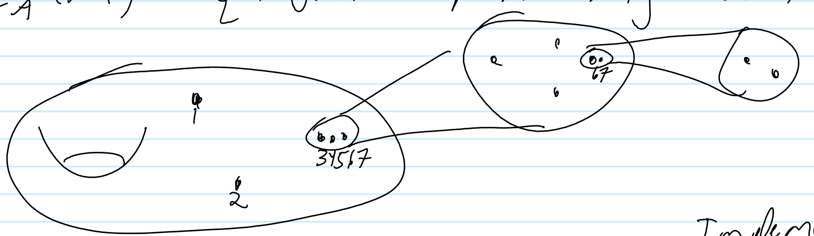
refs: Bott & Taubes, Thurston D

$$Z_1(\gamma) = \sum_{D\in\{\ \}} \frac{D}{|\mathrm{Aut}(D)|}\int_{C_D(\mathbb{R}^3,\gamma)\subset (S^1)^{V_s(D)}\times(\mathbb{R}^3)^{V_i(D)}} \prod_{e\in E(D)} \Phi_e^* w \in \mathcal{D}^{-1}$$

on board

Let M be a d-manifold & A a finite set

$C_A^\circ(M) := \{\text{injections } p: A\to M\}$ dim $C_A^\circ(M) = |A|\cdot d$



Implementation:

$$C_A(M) := \coprod_{\{A_1,\dots,A_k\},\ A=\cup A_\alpha} \left\{ \left(p_\alpha\in M, c_\alpha \in \tilde C_{A_\alpha}(T_{p_\alpha}M)\right)_{\alpha=1}^k : p_\alpha\ne p_\beta \text{ for } \alpha\ne\beta\right\}$$

where if V is a vector space and A is a singleton, $\tilde C_A(V) := \{\text{a point}\}$ and if $|A|\ge 2$,

$$\tilde C_A(V) := \coprod_{\substack{\{A_1,\dots,A_k\}\\ A=\cup A_\alpha;\ k\ge2}} \left\{\left(v_\alpha\in V, c_\alpha\in\tilde C_{A_\alpha}(T_{v_\alpha}V)\right)_{\alpha=1}^k : v_\alpha\ne v_\beta \text{ for }\alpha\ne\beta\right\}\Big/ \text{translations and dilations.}$$

acting on the v's.

"big cell" is of dim $= d\cdot|A| - d - 1$

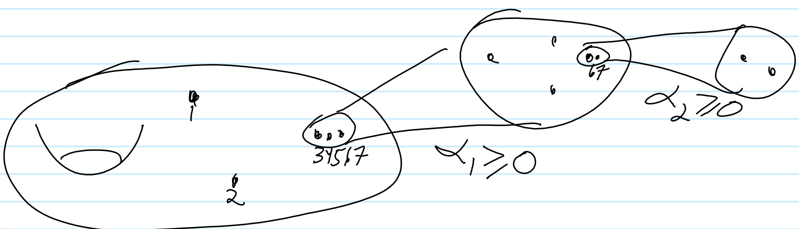
$\Rightarrow$ every "grouping" loses one dimension.

<u>Def</u> A d-manifold w/ corners (modeled on $\mathbb{R}^d_{+k}$)

<u>Thm</u> $C_A(M)$ is a $d\cdot|A|$-manifold w/ corners &

$$\partial M = \coprod_{A'\subset A,\ |A'|\ge2} \left\{(p,c):\ p\in C^o_{A/A'}(M),\ c\in\tilde C_{A'}(T_{p_{A'}}M)\right\}.$$

A complete proof would be Hell on Earth, and I'm not sure it was ever written. Sketch:



Blunders:
1. Is a manifold w/ corners automatically a mfd w/ boundary.
2. The description of the tangent space on the left was lacking.

done line

ways' lacking.

done line

Thm 1. M compact $\Rightarrow C_A(M)$ compact.

2. singletons & doubletons.

3. $B \subset A \Rightarrow \exists\, p_B : C_A(M) \to C_B(M)$. In particular,

$$\exists\ \phi_{ij} : C_A(\mathbb{R}^n) \longrightarrow C_{\{i,j\}}(\mathbb{R}^n) \sim S^{n-1}$$

4. If $f: M \to N$ is a smooth embedding,

$$\exists\ f_* : C_A(M) \longrightarrow C_A(N)$$

Skip section in handout about $C_D(M)$; just write $C_D^o(M) := \{p\colon A \to M\colon\ p(a_0) \neq p(a_1) \text{ whenever } a_0 \overset{D}{\text{—}} a_1\}$

Definition 9. Write $S^n = \mathbb{R}^n \cup \{\infty\}$ and set $\bar{C}_A(\mathbb{R}^n) := \left\{c \in \tilde{C}_{A \cup \{\infty\}}(S^n)\colon\ p_\infty(c) = \infty\right\}$.

Theorem 10. $\bar{C}_A(\mathbb{R}^n)$ is a compact manifold with corners and the direction maps $\phi_{ij}\colon \bar{C}_A(\mathbb{R}^n) \to S^{n-1}$ remain well-defined.

Finally, given $\gamma\colon S^1 \to \mathbb{R}^3$ and disjoint finite sets A and B, we set

$$C^\gamma_{A,B} := \left\{(c', c)\colon\ c' \in C_A(S^1),\ c \in \bar{C}_{A \cup B}(\mathbb{R}^3),\ \gamma_*(c') = p_A(c)\right\}$$

(and similarly C^γ_D for appropriate graphs D). The obvious variants of the theorems remain valid.