Monday-8 AKT on 140303: Lie Algebraic Weight Systems

February-26-14 9:28 AM

HW6 on web!

Reminders $\mathfrak{g}$: metrized f.d. Lie algebra.

R: Representation thereoff.

on board

Thm $\exists\ W_{\mathfrak{g},R}: \mathcal{A} \longrightarrow \mathbb{F}$

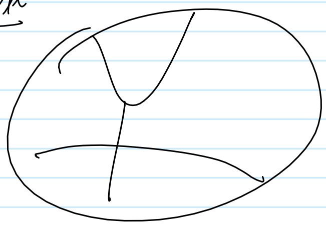
$\mathfrak{g} = \langle X_a \rangle_{a=1}^{\dim \mathfrak{g}}$ $\quad R = \langle e_\alpha \rangle_{\alpha=1}^{\dim R}$

$[X_a, X_b] = f_{ab}^c X_c \quad \langle X_a, X_b \rangle = t_{ab} \quad t_{ab} t^{bc} = \delta_a^c$

$X_a e_\alpha = \rho_{a\alpha}^\beta e_\beta$

$f_{abc} = \langle [X_a, X_b], X_c \rangle = f_{ab}^d t_{dc}$

Example



Thm This is well defined on diag.

Pf 1. Phys way.

2. Tens. calc. way.

3. hands on.

Thm Satisfies AS, IHX, STU.

done line.

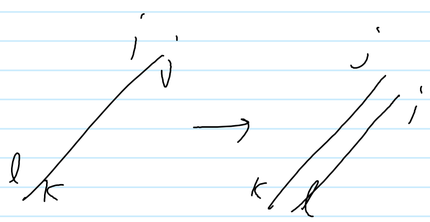
The gl_N calculation $\quad a \leftrightarrow (ij)$

$X_{ij} = \begin{array}{cc} & j \\ i & \left(\quad \right) \end{array}$

$X_{ij} X_{kl} = \delta_{jk} X_{il}$

$[X_{ij}, X_{kl}] = \delta_{jk} X_{il} - \delta_{il} X_{kj}$

$t_{(ij)(kl)} = tr\, X_{ij} X_{kl} = \delta_{jk} \delta_{il}$

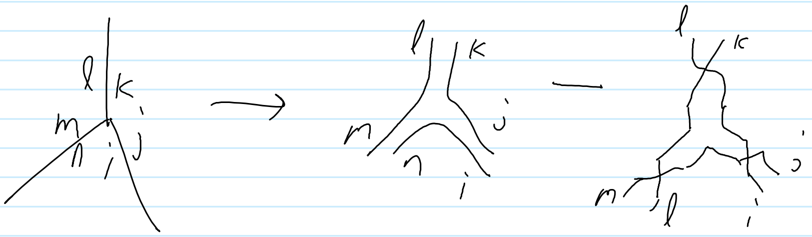


$t^{(ij)(kl)} =$ same; indeed

$t_{ij,kl} t^{kl,mn} = \sum_{k,l} \delta_{jk} \delta_{il} \delta^{kn} \delta^{lm} = \delta_{jn} \delta_{im}$

$F_{ij,kl,mn} = \langle [X_{ij}, X_{kl}], X_{mn} \rangle = \langle \delta_{jk} X_{il} - \delta_{il} X_{kj}, X_{mn} \rangle$

$$= \delta_{jk}\delta_{lm}\delta_{in} - \delta_{il}\delta_{jm}\delta_{kn}$$



$$r^{l}_{(ij)k} = \delta_{jk}\delta^{l}_{i}$$