Friday-4 AKT on 140131: Scratch

January-28-14 9:15 AM

on board

$\Lambda: V \to V^*$, symmetric & P.D., $\varphi_1, \varphi_2 \in V^*$, $Z := \int_{x\in V} dx\, e^{-\frac{1}{2} v\cdot\Lambda v}$,

Then $Z^{-1} \int_{x\in V} dx\, e^{-\frac{1}{2} v\cdot\Lambda v} \varphi_1(v)\varphi_2(v) = \varphi_1 \Lambda^{-1} \varphi_2$

$$Z(\gamma_1,\gamma_2) := Z^{-1} \int_{A\in\Omega^1(\mathbb{R}^3)} \mathcal{D}A\, e^{\frac{i}{4\pi}\int_{\mathbb{R}^3} A\wedge dA} \cdot \int_{\gamma_1} A \cdot \int_{\gamma_2} A = C\langle \gamma_1 | d^{-1} | \gamma_2 \rangle$$

$$\Big\downarrow$$

$$Z^{-1} \int_{\substack{A\in\Omega^1(\mathbb{R}^3) \\ A\perp\{df\} \iff d^*A=0}} \underbrace{\qquad\text{same}\qquad} \int_{\gamma_1} A \cdot \int_{\gamma_2} A$$

now γ_i must be closed ?

$$\{A\in\Omega^1(\mathbb{R}^3) : d^*A = 0\}^* = \Omega^2(\mathbb{R}^3)/$$

$(\ker L)^* =$

$V \xrightarrow{L} W \qquad W \subset V \qquad W^* = V^*/W^\perp$

B, d^*A

More precisely, $\Lambda : \{1\text{-forms}\} \to \langle \text{Currents} \rangle \quad j$

$\searrow^{d}$ $\{2\text{-forms}\}$ (= Currents) $\qquad \int_D \alpha = \int_D j\cdot\vec{n}$

$\Lambda^{-1} \sim d^{-1} : \langle \text{currents} \rangle \to \Omega^1$

$$j \longrightarrow \text{the 1-form } \lambda \text{ s.t.}$$

$$\int_{\partial D} \lambda = \int_D d\lambda = \int_D j \cdot \vec{n}$$

$\Lambda^{-1}\gamma_2$: The 1-form λ whose integral around a small loop ∂D is 1 iff γ_2 pierces D positively.

$$\langle \gamma_1 | \Lambda^{-1} | \gamma_2 \rangle = l(\gamma_1, \gamma_2) \quad !$$

<u>Claim</u> If $\alpha \in \Omega^2(\mathbb{R}^3)$ & $d\alpha = 0$, consider

$\phi : \mathbb{R}^3_x \times \mathbb{R}^3_y \longrightarrow S^2 \quad (x,y) \longmapsto \frac{x-y}{\| \; \|}$ $\quad w$ vol. on S^2

$\pi_{x,y} : \mathbb{R}^3_x \times \mathbb{R}^3_y \longrightarrow \mathbb{R}^3_x, \mathbb{R}^3_y$ projections

and set

$$\sigma = \int_{\mathbb{R}^3_y} \left(\phi^*(w) \wedge \pi_y^* \alpha \right) \in \Omega^1(\mathbb{R}^3_x)$$

Then $d\sigma = \alpha$

"a formula for d^{-1}"