Agenda. Three courses on just one theorem: With $\mathcal{K}$ the set of knots and $\mathcal{A}$ something naturally associated to knots and quite related to Lie algebras, there exists an expansion $Z\colon \mathcal{K} \to \mathcal{A}$.

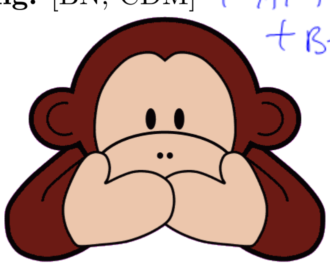
What's "an expansion"? Given a "ring" K and an ideal $I \subset K$, set
$$A := I^0/I^1 \oplus I^1/I^2 \oplus I^2/I^3 \oplus \cdots.$$
An expansion is $Z\colon K \to A$ such that if $\gamma \in I^m$, then
$$Z(\gamma) = (0, 0, \ldots, 0, \gamma/I^{m+1}, *, *, \ldots).$$

Example. Let $K = C^\infty(\mathbb{R}^n)$ be smooth functions on $\mathbb{R}^n$, and $I := \{f \in K\colon f(0) = 0\}$. Then $I^m = \{f\colon f \text{ vanishes as } |x|^m\}$ and I^m/I^{m+1} is {homogeneous polynomials of degree m} and A is the set of power series. So Z is "a Taylor expansion".
Hence Taylor expansions are vastly general; even knots can be Taylor expanded! And that's what this course is about.
Prerequisites. **1.** Some mathematical maturity. **2.** Differential forms and Stokes' theorem at the level of our first-semester core topology class.

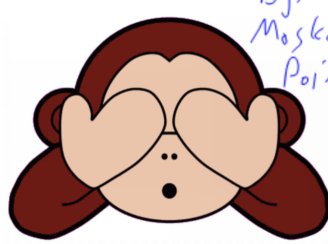
Very Rough 3-Course Plan

some time-trading between the courses is possible

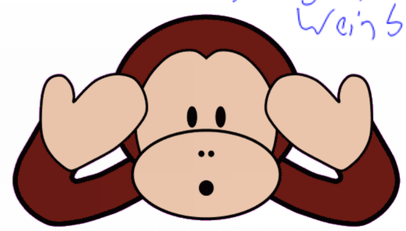
Monday Course. Why is this natural, desired, expected, and hard, from the perspective of knot theory.
1. Course introduction, knots and Reidemeister moves, knot colourings. **2.** The Kauffman bracket and the Jones polynomial (with some programming). **3.** Finite type invariants to weight systems. **4.** The weight system of the Jones polynomial and some other combinatorial weight systems. **5.** The Fundamental Theorem, universal finite type invariants, expansions. **6.** $\mathcal{A}$ as a bi-algebra. **7.** The bracket-rise theorem. **8.** Lie algebras and Lie-algebraic weight systems. **9.** Evaluation for $gl(N)$. **10.** Knotted trivalent graphs and algebraic knot theory. **11.** Expansions for knotted trivalent graphs, associators.
Reading. [BN, CDM] + AKTCFA + B-N Dancso



Wednesday Course. A proof of the theorem using differential geometry and "configuration space integrals".
1. The Gauss linking number combinatorially and as an integral. **2.** The self-linking number and framings. **3.** More on self-linking. **4.** Configuration space integrals and Z_0. **5.** The framing anomaly and Z. **6.** Manifolds with corners and the Fulton-MacPherson compactification. **7.** More on same. **8.** Pushforwards of differential forms and Stokes' theorem for pushforwards. **9.** Proof of the Fundamental Theorem. **10.** Extremal gauge choices. Dylan, Moskovich Poirier



Friday Course. Where did that proof come from? A very gentle introduction to quantum field theory and Feynman diagrams, at imperfect rigour.
1. The Schrödinger equation and path integrals. **2.** What happens to an electron in a quantum harmonic oscillator? **3.** Gaussian integration in $\mathbb{R}^n$. **4.** Feynman diagrams in $\mathbb{R}^n$. **5.** Abelian Chern-Simons theory. **6.** The Gaussian linking number and the self-linking number. **7.** Non Abelian Chern-Simons theory, gauge invariance, and Wilson loops. **8.** Faddeev-Popov gauge fixing and perturbation theory for determinants. **9.** Berezin integration. **10.** BRST. **11.** The Axelrod-Singer change of variables and the reduction to configuration space integrals.
Reading. [Wi] AS, Polyak, Weinburg



The Details.

Classes. Mondays and Wednesdays at 11:10-12:00 and Fridays at 10:10-11:00 at Bahen 6180.

Instructor. Dror Bar-Natan, drorbn@math.toronto.edu, `http://www.math.toronto.edu/~drorbn/`, Bahen 6178, 416-946-5438. Office hours: by appointment or by opportunity.

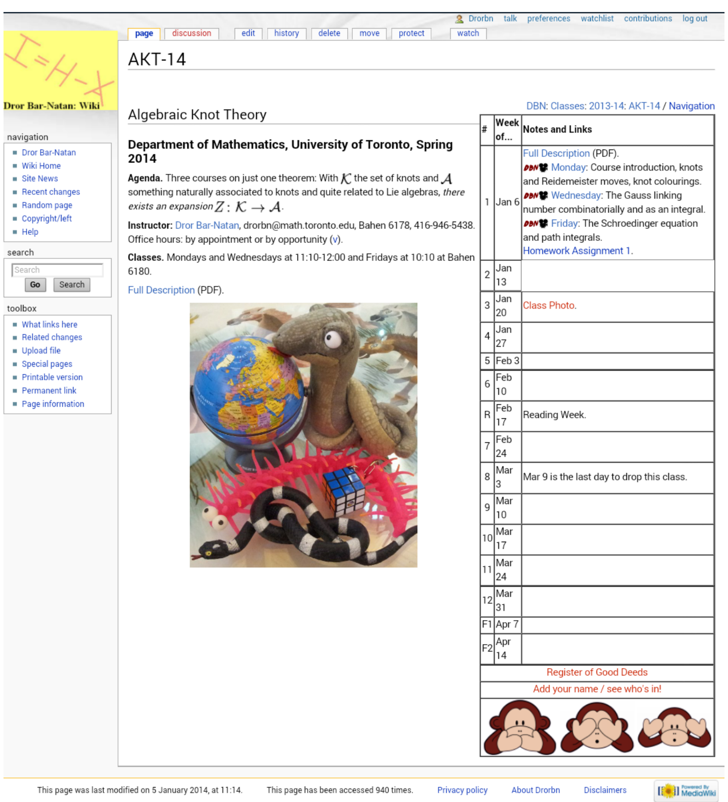
Class Photo. To help me learn your names, ~~I~~ we will take a class photo on Wednesday of the third week of classes. I will post the picture on the class' web site and you will be encouraged to identify yourself on the Class Photo page (`http://drorbn.net/?title=AKT-14/Class_Photo`) of the class' wiki.

Videos and Wiki. We will videotape all classes and the course's web site will be centered around these videos. I have set up a system (see below) that allows anyone signed-up to index and annotate these videos on a wiki (as in Wikipedia), and allows for the inclusion and linking of other pages and further material to this wiki.
Anyone signed-up can, is welcome and is encouraged to edit and add to the class' web site. In particular, students can post video annotations, notes, comments, pictures, solution

to open problems, whatever. Some rules, though —

- This wiki is a part of my (Dror's) academic web page. All postings on it must be class-related (or related to one of the other projects I'm involved with).
- Criticism is fine, but no insults or foul language, please.
- I (Dror) will allow myself to exercise editorial control, when necessary.
- The titles of all pages related to this class should contain and preferably begin with the string "`AKT-14`", just like the title the classes' main page.
- For most `AKT-14` pages, it is a good idea to put a line containing only the string `{{AKT-14/Navigation}}` at the top of the page. This template inserts the class' "navigation panel" at the top right of the page.
- To edit the navigation panel itself, click on the word "Navigation" on the upper right of the panel. Use caution! Such edits affect many other pages! Note that due to page-caching, such edits take some time to propagate to the pages that include the navigation panel. To force immediate propagation to a given page, reload that page with the string "`&action=purge`" (meaning: "purge cached version") appended to the page's URL.
- Neatness matters! Material that is posted in an appealing manner will be read more, and thus will be more useful.
- Some further editing help is available at `http://drorbn.net/?title=Help:Contents`.

Wiki Sign-Up. Email me with you full name, email address and preferred userid if you need an account on the class wiki.



Warnings. The videos will be kept on my personal server, which has limited bandwidth. That server is not professionally maintained, and it may be down for arbitrary reasons at arbitrary times, and I may or may not rush to fix it. The default video player may or may not be compatible with your computer / browser. I will deal with compatibility issues iff I'll have the time and the mood. If you cannot view a certain video, try the "download" link on the upper right of the player rectangle. If even that fails, you may download stuff straight from `http://drorbn.net/dbnvp/ogg/` (look for filenames of the form "`AKT-14mmdd_www.ogg` ", with `mm`=month, `dd`=day, and `www`=width). An external player that plays almost anything on anything is VLC, at `http://www.videolan.org/`.

Homework assignments will be jointly written, usually on Fridays, usually they will be assigned on Mondays, and usually be due on the following Monday. There will be about 11 assignments; your HW mark will be the average of your best 6 assignments. Late assignments will be marked down by 1% per day.

Semi-Final grade. The higher of 70% HW and 30% Final Examination, or 20% HW and 80% Final Examination.

Good deeds. You will be able to earn "good deed" points throughout the term. You may earn up to 80 good deed points for writing a book-quality open-source and copyleft exposition of one of the three classes. More realistically, for lively participation in and markup of the class wiki, you may receive up to about 30 good deed points.

Final Grade. If you earn $0 \leq \gamma \leq 80$ good deed points during the term, and your semi-final grade is σ, your final grade will be $\gamma+(100-\gamma)\sigma/100$. This can be 100 even with $\gamma=0$, yet with $\gamma=80$, it will be 90 even with $\sigma=50$.

Dror's Open Notebook for this class is at `http://drorbn.net/AcademicPensieve/Classes/14-1350-AKT/`. Use at your own risk.

References.

[BN] D. Bar-Natan, *On the Vassiliev knot invariants,* Topology **34** (1995) 423–472.

[CDM] S. Chmutov, S. Duzhin, and J. Mostovoy, *Introduction to Vassiliev Knot Invariants,* Cambridge University Press, Cambridge UK, 2012.

[Wi] E. Witten, *Quantum field theory and the Jones polynomial,* Commun. Math. Phys. **121** (1989) 351–399.