http://drorbn.net/?title=AKT-14
Dror Bar-Natan: Classes: 2013-14: 1350-AKT:

About This Class

Three courses on just one **Theorem:** With $\mathcal{K}$ the set of knots and $\mathcal{A}$ something naturally associated to knots and quite related to Lie algebras, there exists an expansion $Z\colon \mathcal{K}(\uparrow) \to \mathcal{A}(\uparrow)$.

Prerequisites. 1. Some mathematical maturity. **2.** Differential forms and Stokes' theorem at the level of our first-semester core topology class.

What's "an expansion"? Given a "ring" K and an ideal $I \subset K$, set

$$A := I^0/I^1 \oplus I^1/I^2 \oplus I^2/I^3 \oplus \cdots .$$

An expansion is $Z\colon K \to A$ such that if $\gamma \in I^m$, then

$$Z(\gamma) = (0, 0, \ldots, 0, \gamma/I^{m+1}, *, *, \ldots).$$

Example. Let $K = C^\infty(\mathbb{R}^n)$ be smooth functions on $\mathbb{R}^n$, and $I := \{f \in K\colon f(0) = 0\}$. Then $I^m = \{f\colon f \text{ vanishes as } |x|^m\}$ and I^m/I^{m+1} is $\{\text{homogeneous polynomials of degree } m\}$ and A is the set of power series. So Z is "a Taylor expansion".

Hence Taylor expansions are vastly general; even knots can be Taylor expanded! And that's what this course is about.

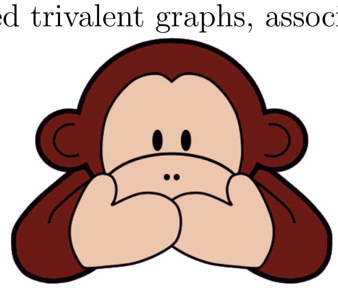
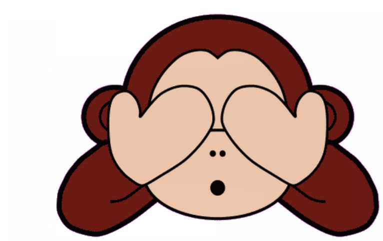
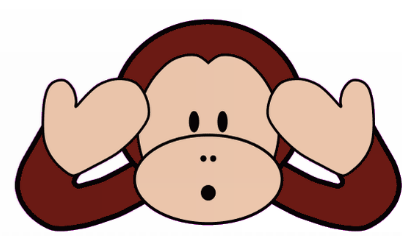
Very Rough 3-Course Plan

some time-trading between the courses is possible

Monday Course. Why is this natural, desired, expected, and hard, from the perspective of knot theory.
1. Course introduction, knots and Reidemeister moves, knot colourings. **2.** The Kauffman bracket and the Jones polynomial (with some programming). **3.** Finite type invariants to weight systems. **4.** The weight system of the Jones polynomial and some other combinatorial weight systems. **5.** The Fundamental Theorem, universal finite type invariants, expansions. **6.** $\mathcal{A}$ as a bi-algebra. **7.** The bracket-rise theorem. **8.** Lie algebras and Lie-algebraic weight systems. **9.** Evaluation for $gl(N)$. **10.** Knotted trivalent graphs and algebraic knot theory. **11.** Expansions for knotted trivalent graphs, associators.

Wednesday Course. A proof of the theorem using differential geometry and "configuration space integrals".
1. The Gauss linking number combinatorially and as an integral. **2.** The self-linking number and framings. **3.** More on self-linking. **4.** Configuration space integrals and Z_0. **5.** The framing anomaly and Z. **6.** Manifolds with corners and the Fulton-MacPherson compactification. **7.** More on same. **8.** Pushforwards of differential forms and Stokes' theorem for pushforwards. **9.** Proof of the Fundamental Theorem. **10.** Extremal gauge choices.

Friday Course. Where did that proof come from? A very gentle introduction to quantum field theory and Feynman diagrams., at imperfect rigour.
1. The Schrödinger equation and path integrals. **2.** What happens to an electron in a quantum harmonic oscillator? **3.** Gaussian integration in $\mathbb{R}^n$. **4.** Feynman diagrams in $\mathbb{R}^n$. **5.** Abelian Chern-Simons theory. **6.** The Gaussian linking number and the self-linking number. **7.** Non Abelian Chern-Simons theory, gauge invariance, and Wilson loops. **8.** Faddeev-Popov gauge fixing and perturbation theory for determinants. **9.** Berezin integration. **10.** BRST. **11.** The Axelrod-Singer change of variables and the reduction to configuration space integrals.

The Details.

Classes. Mondays and Wednesdays at 11:10-12:00 and Fridays at 10:10 at Bahen 6180.

Instructor. Dror Bar-Natan, drorbn@math.toronto.edu, `http://www.math.toronto.edu/~drorbn/`, Bahen 6178, 416-946-5438. Office hours by appointment.

Videos and Wiki. We will videotape all classes and the course's web site will be centered around these videos. I have set up a system that allows anyone signed-up to index and annotate these videos on a wiki (as in Wikipedia), and allows for the inclusion and linking of other pages and further material to this wiki.

Anyone signed-up can, is welcome and is encouraged to edit and add to the class' web site. In particular, students can post video annotations, notes, comments, pictures, solution to open problems, whatever. Some rules, though —

- This wiki is a part of my (Dror's) academic web page. All postings on it must be class-related (or related to one of the other projects I'm involved with).
- Criticism is fine, but no insults or foul language, please.
- I (Dror) will allow myself to exercise editorial control, when necessary.
- The titles of all pages related to this class should contain and preferably begin with the string "AKT-14", just like the title the classes' main page.

Email me with you full name, email address and preferred userid if you need an account on the class wiki. Some further editing help is available at `http://drorbn.net/?title=Help:Contents`.

HW: jointly written usually every Friday, usually assigned on Monday, usually due the following Monday.

There will be about 11 HW assignments, your HW mark will be "average of best 6"; late assignments will be marked down by 1% per day.

Final grade: Higher of 70% HW & 30% Final or 20% HW & 80% Final.

Good deeds up to 80 for writing a book-quality open source & copyleft exposition of one of the 3 classes. More realistically, for lively participation & markup of the class wiki, you may receive up to about 30 good deed points.