Agenda. Three courses on just one theorem: With $\mathcal{K}$ the set of knots and $\mathcal{A}$ something naturally associated to knots and quite related to Lie algebras, there exists an expansion $Z \colon \mathcal{K} \to \mathcal{A}$.

What's "an expansion"? Given a "ring" K and an ideal $I \subset K$, set

$$A := I^0/I^1 \oplus I^1/I^2 \oplus I^2/I^3 \oplus \cdots .$$

An expansion is $Z \colon K \to A$ such that if $\gamma \in I^m$, then

$$Z(\gamma) = (0, 0, \ldots, 0, \gamma/I^{m+1}, *, *, \ldots).$$

Example. Let $K = C^\infty(\mathbb{R}^n)$ be smooth functions on $\mathbb{R}^n$, and $I := \{f \in K \colon f(0) = 0\}$. Then $I^m = \{f \colon f \text{ vanishes as } |x|^m\}$ and I^m/I^{m+1} is {homogeneous polynomials of degree m} and A is the set of power series. So Z is "a Taylor expansion". Hence Taylor expansions are vastly general; even knots can be Taylor expanded! And that's what this course is about.

Prerequisites. 1. Some mathematical maturity. **2.** Differential forms and Stokes' theorem at the level of our first-semester core topology class.

Very Rough 3-Course Plan
some time-trading between the courses is possible

Monday Course. Why is this natural, desired, expected, and hard, from the perspective of knot theory.
1. Course introduction, knots and Reidemeister moves, knot colourings. **2.** The Kauffman bracket and the Jones polynomial (with some programming). **3.** Finite type invariants, some examples. **4.** Weight systems. **5.** 4T and the Fundamental Theorem, universal finite type invariants. **6.** The bracket-rise theorem. **7.** Lie algebras and Lie-algebraic weight systems. **8.** Evaluation for $gl(N)$. **9.** $\mathcal{A}$ as a bialgebra. **10.** Planar algebras and expansions. **11.** Knotted trivalent graphs and algebraic knot theory. **12.** Expansions for knotted trivalent graphs, associators.
Reading. [BN1, CDM, BND].

Wednesday Course. A proof of the theorem using differential geometry and "configuration space integrals".
**1.** The Gauss linking number combinatorially and as an integral. **2.** The self-linking number and swaddling maps. **3.** The self-linking number and framings. **4.** More on framings and swaddling maps, configuration space integrals. **5.** The Fulton-MacPherson compactification. **6.** More on the same, graph homology. **7.** The framing anomaly and Z. **8.** Pushforwards of differential forms and Stokes' theorem for pushforwards. **9.** Proof of the Fundamental Theorem. **10.** Extremal gauge choices.
Reading. [Mo, BT, Th, Po].

Friday Course. Where did that proof come from? A very gentle introduction to quantum field theory and Feynman diagrams, at imperfect rigour.
1. The Schrödinger equation and path integrals. **2.** What happens to an electron in a quantum harmonic oscillator? **3.** Gaussian integration in $\mathbb{R}^n$. **4.** Abelian Chern-Simons theory. **5.** The Gaussian linking number and the self-linking number. **6.** Non Abelian Chern-Simons theory, gauge invariance, and Wilson loops. **7.** Feynman diagrams in $\mathbb{R}^n$. **8.** Faddeev-Popov gauge fixing and perturbation theory for determinants. **9.** Berezin integration. **10.** BRST. **11.** The Axelrod-Singer change of variables and the reduction to configuration space integrals.
Reading. [Wi, BN2, AS, We].
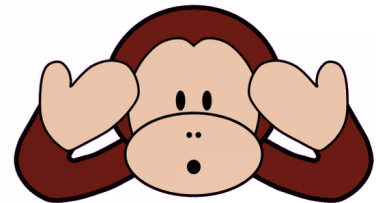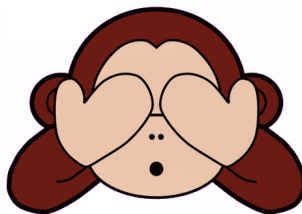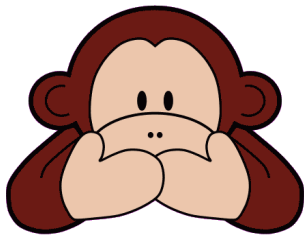
Each course is but the tip of a huge iceberg. . .

The Details.

Classes. MWF at 10:10-11:00; Mondays and Fridays at Bahen 6180 but Wednesdays at Huron 1018. There will also be a "HW meeting", covering no new material, on Fridays at 6:10PM at Bahen 6180.

Instructor. Dror Bar-Natan, drorbn@math.toronto.edu, `http://www.math.toronto.edu/~drorbn/`, Bahen 6178, 416-946-5438. Office hours: by appointment or by opportunity.

Class Photo. To help me learn your names, I will take a class photo on Friday of the third week of classes. I will post the picture on the class' web site and you will be encouraged to identify yourself on the Class Photo page (`http://drorbn.net/?title=AKT-14/Class_Photo`) of the class' wiki.

Videos and Wiki. We will videotape all classes and the course's web site will be centered around these videos. I have set up a system (see below) that allows anyone signed-up to index and annotate these videos on a wiki (as in Wikipedia), and allows for the inclusion and linking of other pages and further material to this wiki.

Anyone signed-up can, is welcome and is encouraged to edit and add to the class' web site. In particular, students can post video annotations, notes, comments, pictures, solution to open problems, whatever. Some rules, though —

• This wiki is a part of my (Dror's) academic web page. All postings on it must be class-related (or related to one of the other projects I'm involved with).

• Criticism is fine, but no insults or foul language, please.

• I (Dror) will allow myself to exercise editorial control, when necessary.

• The titles of all pages related to this class should contain and preferably begin with the string "`AKT-14`", just like the title the classes' main page.

• For most `AKT-14` pages, it is a good idea to put a line containing only the string `{{AKT-14/Navigation}}` at the top of the page. This template inserts the class' "navigation panel" at the top right of the page.

• To edit the navigation panel itself, click on the word "Navigation" on the upper right of the panel. Use caution! Such edits affect many other pages! Note that due to page-caching, such edits take some time to propagate to the pages that include the navigation panel. To force immediate propagation to a given page, reload that page with the string "`&action=purge`" (meaning: "purge cached ver-

sion") appended to the page's URL.

• Neatness matters! Material that is posted in an appealing manner will be read more, and thus will be more useful.

• Some further editing help is available at `http://drorbn.net/?title=Help:Contents`.

Wiki Sign-Up. Email me with you full name, email address and preferred userid if you need an account on the class wiki.

Warnings. The videos will be kept on my personal server, which has limited bandwidth. That server is not professionally maintained, and it may be down for arbitrary reasons at arbitrary times, and I may or may not rush to fix it. The default video player may or may not be compatible with your computer / browser. I will deal with compatibility issues iff I'll have the time and the mood. If you cannot view a certain video, try the "download" link on the upper right of the player rectangle. If even that fails, you may download stuff straight from `http://drorbn.net/dbnvp/ogg/` (look for filenames of the form "`AKT-14mmdd_www.ogg`", with `mm`=month, `dd`=day, and `www`=width). An external player that plays almost anything on anything is VLC, at `http://www.videolan.org/`.

Homework assignments will usually be jointly written, usually on the Friday HW meetings, usually they will be assigned on Mondays, and usually be due on the following Monday. There will be about 11 assignments; your HW mark will be the average of your best 6 assignments. Late assignments will be marked down by 1% per day.

Semi-Final grade. The higher of 70% HW and 30% Final Examination, or 20% HW and 80% Final Examination.

Good deeds. You will be able to earn "good deed" points throughout the term. You may earn up to 80 good deed points for writing a book-quality open-source and copyleft exposition of one of the three classes. More realistically, for lively participation in and markup of the class wiki, you may receive up to about 30 good deed points.

Final Grade. If you earn $0 \le \gamma \le 80$ good deed points during the term, and your semi-final grade is σ, your final grade will be $\gamma + (100 - \gamma)\sigma/100$. This can be 100 even with $\gamma = 0$, yet with $\gamma = 80$, it will be 90 even with $\sigma = 50$.

Dror's Open Notebook for this class is at `http://drorbn.net/AcademicPensieve/Classes/14-1350-AKT/`. Use at your own risk.

References.

[AS] S. Axelrod and I. M. Singer, *Chern-Simons perturbation theory,* Proceedings of the XXth International Conference on Differential Geometric Methods in Theoretical Physics, (New York, 1991), 3–45, World Sci. 1992, arXiv:hep-th/9110056.

[BN1] D. Bar-Natan, *On the Vassiliev knot invariants,* Topology **34** (1995) 423–472.

[BN2] D. Bar-Natan, *Perturbative Chern-Simons Theory,* J. of Knot Theory and its Ramifications, **4-4** (1995) 503–548.

[BND] D. Bar-Natan and Z. Dancso, *Homomorphic expansions for knotted trivalent graphs*, J. of Knot Theory and its Ramifications **22-1** (2013), arXiv:1103.1896

[BT] R. Bott and C. Taubes, *On the self-linking of knots,* Jour. Math. Phys. **35** (1994).

[CDM] S. Chmutov, S. Duzhin, and J. Mostovoy, *Introduction to Vassiliev Knot Invariants,* Cambridge University Press, Cambridge UK, 2012.

[Mo] D. Moskovich, *Framing and the self-linking integral,* arXiv:math.QA/0211223.

[Po] S. Poirier, *The configuration space integral for links and tangles in* $\mathbb{R}^3$*,* Algebraic and Geometric Topology, **2-40** (2002) 1001-1050, arXiv:math.GT/0005085.

[Th] D. Thurston, *Integral expressions for the Vassiliev knot invariants,* Harvard University senior thesis, April 1995, arXiv:math.QA/9901110.

[We] S. Weinberg, *The Quantum Theory of Fields,* Cambridge University Press, 1995.

[Wi] E. Witten, *Quantum field theory and the Jones polynomial,* Commun. Math. Phys. **121** (1989) 351–399.

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AKT-14

$I = H - X$

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Algebraic Knot Theory

DBN: Classes: 2013-14: AKT-14 / Navigation

Department of Mathematics, University of Toronto, Spring 2014

Agenda. Three courses on just one theorem: With $\mathcal{K}$ the set of knots and $\mathcal{A}$ something naturally associated to knots and quite related to Lie algebras, *there exists an expansion* $Z: \mathcal{K} \to \mathcal{A}$.

Instructor: Dror Bar-Natan, drorbn@math.toronto.edu, Bahen 6178, 416-946-5438. Office hours: by appointment or by opportunity (v).

Classes. Mondays and Wednesdays at 11:10-12:00 and Fridays at 10:10-11:00 at Bahen 6180.

About This Class (PDF).

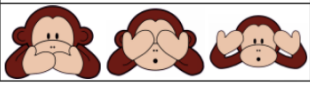| #	Week of...	Notes and Links
1	Jan 6	About This Class (PDF). *DBN* Monday: Course introduction, knots and Reidemeister moves, knot colourings. *DBN* Wednesday: The Gauss linking number combinatorially and as an integral. *DBN* Friday: The Schroedinger equation and path integrals. Homework Assignment 1.
2	Jan 13	
3	Jan 20	Class Photo.
4	Jan 27	
5	Feb 3	
6	Feb 10	
R	Feb 17	Reading Week.
7	Feb 24	
8	Mar 3	Mar 9 is the last day to drop this class.
9	Mar 10	
10	Mar 17	
11	Mar 24	
12	Mar 31	
F1	Apr 7	
F2	Apr 14	
		Register of Good Deeds
		Add your name / see who's in!

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click to add a wiki comment on minute 0:15:44 of this video.

Then hit "refresh" and your new comment will appear on the right panel, about here

Edit the title of this specific video; it will be pinked-out in the table of content.

click to jump to the "section 2.1" video.

Left panel

Right panel

Click "add" to add a wiki comment on minute 15:04

This blackboard shot was taken at minute 15:04 of the talk. This more or less means that the discussion of its content ended at that minute.

Click the blackboard shot to enlarge it (opens in another tab)

click "edit" to edit the wiki comment for minute 19:30

click the 19:30 link to jump to minute 19:30 of the video.

Drag red line left or right to change panel sizes.

Wiki: To add/modify wiki comments you must have an account on Dror's wiki. Email drorbn@math.toronto.edu to ask for one, if you don't already have one.