

The inclusions

$$\text{UFD} \stackrel{1}{\not\subseteq} \text{PID} \stackrel{2}{\not\subseteq} \text{Euc}$$

are strict.

1. Many examples; especially polynomial rings in several variables and $\mathbb{Z}[x]$. (In general, $R \text{ UFD} \Rightarrow R[x] \text{ UFD}$).
2. Examples are hard. The easiest seems to be $\mathbb{Z}\left[\frac{1+\sqrt{-19}}{2}\right]$.

A sequence of exercises leading to a proof is in eprints/Bergman:

Math 250A, G. Bergman, 2002

A principal ideal domain that is not Euclidean
developed as a series of exercises