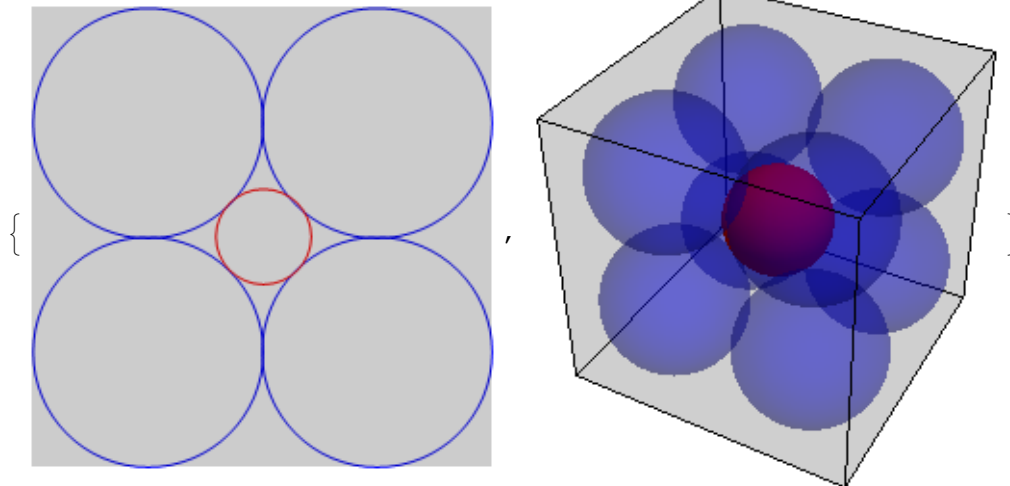


Dror Bar-Natan: Classes: 2014-15: Math 1100 Algebra I:

A riddle from MathOverflow, <http://mathoverflow.net/questions/5372/dimension-leaps/5483#5483>: Put 2^n unit balls centered at the points $(\pm 1, \pm 1, \dots, \pm 1)$ in R^n ; they are all bound in the cube $C=[-2, 2]^n$. Let B be the maximal ball centered at 0 and not intersecting with any of the previous balls. As $n \rightarrow \infty$, what's the limit of $\text{Vol}(B)/\text{Vol}(C)$?

```
Rasterize /@ {
  Graphics[{
    Blue, Circle[List@@#, 1] & /@ Flatten[Outer[Center, {-1, 1}, {-1, 1}]],
    Red, Circle[{0, 0}, Sqrt[2] - 1],
    Black, Opacity[0.2], Rectangle[{-2, -2}, {2, 2}]
  }, ImageSize -> 250],
  Graphics3D[{
    Blue, Opacity[0.3],
    Sphere[List@@#, 1] & /@ Flatten[Outer[Center, {-1, 1}, {-1, 1}, {-1, 1}]],
    Red, Opacity[1], Sphere[{0, 0, 0}, Sqrt[3] - 1],
    Black, Opacity[0.1], Cuboid[{-2, -2, -2}, {2, 2, 2}]
  }, Boxed -> False, ImageSize -> 250]
}
```



```
WolframAlpha["Volume of n-sphere", "Result"]
```

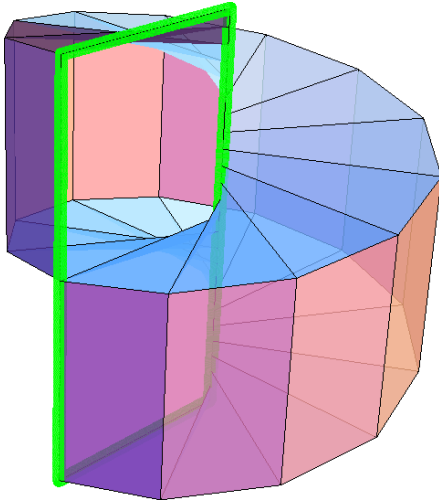
$$\frac{2 \pi^{n/2} r^n}{n \Gamma(\frac{n}{2})} \approx \frac{2 \times 3.14159^{n/2} r^n}{n \Gamma(\frac{n}{2})}$$

(assuming radius r)

```
vr[n_] := (2 \pi^{n/2} (\sqrt{n} - 1)^n) / (n Gamma[n/2]) / 4^n; {Table[vr[n], {n, 2, 10}], vr[100]} // N
```

```
{0.0336883, 0.0256763, 0.0192766, 0.0148324, 0.0117012,
 0.00942996, 0.00773623, 0.0064424, 0.00543354}, 0.0000391445}
```

An image from <http://drorbn.net/AcademicPensieve/2011-12/index.html?im=MobiusStairwell.png>:



Is this stairway shell a 2D manifold (with boundary)? If so, which one is it? If not, why not?

(The classification of surfaces is, for example, at <http://en.wikipedia.org/wiki/Surface>)

Pensieve Header: Riddles for November 24, 2014: a ball squeezed by 2^n balls, and the stairway shell.