

May-30-13  
11:01 AM

**Optimistic Rough Tentative Plan.** Third introduction: Stonehenge. Then perturbative Chern-Simons theory.

Pasted from <<http://www.math.toronto.edu/~drorbn/Talks/Aarhus-1305/>>

1. Go over handout

### From Stonehenge to Witten Skipping all the Details

Opera Meeting on Geometry, Topology and Physics, July 2004  
Dror Bar-Natan, University of Toronto

It is well known that when the Sun rises on midsummer's morning over the "Heel Stone" at Stonehenge, its first rays shine right through the open arms of the horseshoe arrangement. Thus astrological lineups, one of the pillars of modern thought, are much older than the famed Gaussian linking number of two knots.

Recall that the latter is itself an astrological construct: one of the standard ways to compute the Gaussian linking number is to place the two knots in space and then count (with signs) the number of shade points cast on one of the knots by the other knot, with the only lighting coming from some fixed distant star.

The Gaussian linking number  $lk(K_1, K_2) = \frac{1}{2} \sum_{\text{chopsticks}} (\text{signs})$

The Gauss curve slides over a star - Solution: Multiply by a framing-dependent counter-term.

**Theorem.** Modulo Relations,  $Z(K)$  is a knot invariant!

When deforming, catastrophes occur when:  
A plane moves over an intersection point - Solution: Impose IHX.  
An intersection line cuts through the knot - Solution: Impose STU.  
The Gauss curve slides over a star - Solution: Multiply by a framing-dependent counter-term.

**The IHX Relation**

It all is perturbative Chern-Simons-Witten theory:

$$\int_{\text{connections}} DA \text{hol}_k(A) \exp \left[ \frac{ik}{4\pi} \int \text{tr} \left( A \wedge dA + \frac{2}{3} A \wedge A \wedge A \right) \right]$$

$$\rightarrow \sum_{\text{D: Feynman diagrams}} W_k(D) \int \mathcal{E}(D) \rightarrow \sum_{\text{D: Feynman diagrams}} D \int \mathcal{E}(D)$$

Shing-shen Chern      James H Simons

### Knotted Trivalent Graphs, Tetrahedra and Associators

HUI Topology and Geometry Seminar, November 16, 2000  
Dror Bar-Natan

Goal:  $Z: \{\text{knots}\} \rightarrow \{\text{chord diagrams}\} / \text{MT}$  so that

Modulo the relation(s):  $(\text{chord diagram}) = (\text{chord diagram})$

**Claim.** With  $\Phi := Z(A)$ , the above relation becomes equivalent to the Drinfel'd's pentagon of the theory of quasi Hopf algebras.

**Proof.**

Extend to Knotted Trivalent Graphs (KTG's):

Need a new relation:

Easy, powerful moves:

Using moves, KTG is generated by ribbon twists and the tetrahedron:

Further directions:

1. Relations with perturbative Chern-Simons theory.
2. Relations with the theory of 6j symbols
3. Relations with the Turaev-Viro invariants.
4. Can this be used to prove the Witten asymptotics conjecture?
5. Does this extend/improve Drinfel'd's theory of associators?

This handout is at <http://www.ma.huji.ac.il/~drorbn/Talks/HUI-001116>

*Handwritten notes on the handout:*

**Vector space**  
 $dv$ : Liebracket measure on  $V$ .  
 $Q$ : A quadratic form on  $V$ .  
 $L: V \rightarrow V$  is linear.  
 $\text{Combs } I = \int \text{tr}(L^2) \text{vol}$   
 $\text{Combs } II = \int \text{tr}(L^2) \text{vol}$   
 $\text{Combs } III = \int \text{tr}(L^2) \text{vol}$

**In our case,**  
 $Q$  is id, so  $Q^2$  is an integral operator.  
 $P$  is  $\text{Sym}$   
 $H$  is the hatching, itself a sum of  $\text{Sym}$  and  $\text{Asym}$

**The Fourier Transform:**  
 $(F: V \rightarrow V) \rightarrow (F: V \rightarrow V)$   
 via  $F^{-1} = \text{Fourier transform}$

**Simple Facts:**  
 1.  $F^2 = \text{Fourier}$   
 2.  $F^2 = \sqrt{F}$   
 3.  $(F^2)^2 = F^2$   
 where  $Q^2 = (F^2)^2$

**Differentiation and Pairing:**  
 $\partial^2 x^i x^j = \partial^2 x^j x^i$ ; in fact  
 $\{ \partial^2 x^i x^j, \partial^2 x^k x^l \}$   
 $(\partial^2 x^i x^j)^2 = (\partial^2 x^j x^i)^2$  is

**God created the knots, all else in topology is the work of man.**

Leopold Kronecker (modified)

2. Some calls from reality:

a. Nobody has ever written up the Stonehenge story properly.

b. Perturbation theory is more complicated than I made it appear.

c. Stonehenge / Configuration space integrals / CS was never completed for KTGs.

2. There is a KTG invariant but it is only nearly homomorphic

One fix: "splits" for vertices

3. The map  $P_{AB} \rightarrow KTG$

4. Twisting on  $P_{AB}$  & on  $KTG$ ; on knots, associator invariants are independent of the associator.

5. The projectivization machine.