

Optimistic Rough Tentative Plan. Micro-introduction: Knot theory as an excuse and it's all about Taylor. Then KZ, Kontsevich, parenthesized tangles, associators.

Pasted from <<http://www.math.toronto.edu/~drorbn/Talks/Aarhus-1305/>>

0. \mathcal{M} -introduction.

1. \mathcal{A} inherits from \mathcal{K} : $\mathcal{A}(\bigcirc) \cong \mathcal{A}(\uparrow)$ and both are commutative algebras. [Much more to follow]

2. Thm [The fundamental Thm] For every $W \in (\mathcal{A}_m)^*$ there is $V \in \mathcal{K}^*$ s.t. $W = W_V$

claim $\Leftrightarrow \exists Z: \mathcal{K} \rightarrow \hat{\mathcal{A}}$ "an expansion/UT=TI"

$K \in \mathcal{K}_n \Rightarrow Z(K) = D_K + \text{higher order terms.}$

Dror Bar-Natan: Talks / Classes: Aarhus-1305 Day 2 Handout
Clippings from <http://www.math.toronto.edu/~drorbn/CP.html#NAT> and <http://www.math.toronto.edu/~drorbn/Talks/Luminy-9905zeta>

The pentagon:
loosely

The hexagons:
done

not done

3

From http://drorbn.net/index.php?title=07-1352/Class_Notes_for_February_13, though better at <http://www.math.toronto.edu/~drorbn/Talks/Aarhus-1305/KZ.html>

Math 1352 Algebraic Knot Theory - The Knizhnik-Zamolodchikov Connection
Theorem 1. The following is an invariant of braids in $\mathbb{R}^2 \times \mathbb{C}_2$ (Fixed endpoints)

$Z(B) = \int_{\mathcal{P}} \frac{Dp}{(2\pi i)^m} \prod_{i=1}^m \frac{dz_i - dz'_i}{z_i - z'_i}$ in $\mathcal{A}(n) = \langle t^{ij}: |i| \neq |j| \rangle / \begin{cases} t^{ii} = t^{jj} \\ [t^{ij}, t^{jk}] = 0 \\ [t^{ij}, t^{ki}] = 0 \end{cases}$

$\mathcal{P} = \{z_i, z'_i\}$

Formal Connection & Curvature

horizontal axis = "Chord diagrams for braids"

Proof 2. Let $\Gamma: I_3 \times I_4 \rightarrow M, \Phi: I_3 \times I_4 \rightarrow M^*$. By Stokes', $\int_{\partial \Delta^m} \Phi^* \Omega^m - \int_{\partial \Delta^m} \Phi^* \Omega^m = \int_{\partial \Delta^m} d\Phi^* \Omega^m - \int_{\partial \Delta^m} \Phi^* d\Omega^m =: A_m - B_m$. Now $A_m = \sum_{k=1}^m (-1)^k \int_{I_3 \times I_4} \dots$ and $B_m = \int_{I_3 \times I_4} \dots$

Theorem IF $F_u = d\Omega + \Omega \wedge \Omega = 0$, then $\text{hol}_\Omega(\alpha) = \text{Path} \int \Omega = \int_{\Delta^m} \Omega^m$ where $\Omega^m = \prod_{i=1}^m \Omega_i$

The KZ connection. $M = \mathbb{C}^m \setminus \{diagonals\}, A = \mathcal{A}(n)$, and $\Omega = \sum_{i < j} t^{ij} \omega_{ij}$ where $\omega_{ij} = \frac{dz_i - dz_j}{z_i - z_j} \wedge dz_k$

Compute $F_\Omega = d\Omega + \Omega \wedge \Omega$: $d\omega_{ij} = 0$ so $d\Omega = 0$.
 $\Omega \wedge \Omega = \sum_{i < j < k} t^{ij} t^{jk} \omega_{ij} \wedge \omega_{jk} = A + B + C$ where $A = C = 0$ as $[t^{ij}, t^{jk}] = 0$ if $|i| \neq |j| = 2$ or 4 and $B = \sum_{i < j < k} [t^{ij}, t^{jk}] \omega_{ij} \wedge \omega_{jk} + \text{cyclic perms}$.
 $= \sum_{i < j < k} \gamma^{ijk} (\omega_{ij} \wedge \omega_{jk} + \text{cyclic perms}) = 0$ by Annulus identity

Simply take in theorem 2, $\gamma =$ the braid and $\Omega =$ the KZ connection.

Dror Bar-Natan, Feb 13, 2009

3. $f: \mathbb{A}^k \rightarrow \mathbb{A}^k$ by

$$F(D) = \sum_k (-\theta)^k \frac{s^k D}{k!},$$

then $Z(k) = e^{sl(k)\theta} \cdot F(Z(k-1))$

4. A restart: From PaT to $rest \& hex \#$
The right way.

5. What do these things mean in the Lie world?

Not
done