Clippings from Etingof & Kazhdan's arXiv:q-alg/9506005 & arXiv:q-alg/9701038
also see Enriquez' arXiv:math/0101059 and arXiv:math/0212325.

1.1. *Lie bialgebras.*

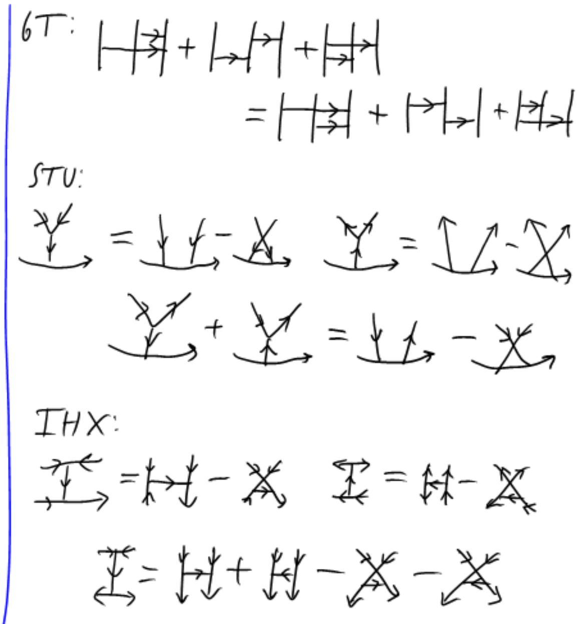
Throughout this paper, k denotes a field of characteristic zero. Let $\mathfrak{a}$ be a Lie algebra over k, and. We δ be a linear map $\delta : \mathfrak{a} \to \mathfrak{a} \otimes \mathfrak{a}$.

Definition.. *One says that the map δ defines a Lie bialgebra structure on $\mathfrak{a}$ if it satisfies two conditions:*

(i) δ is a 1-cocycle of $\mathfrak{a}$ with coefficients in $\mathfrak{a} \otimes \mathfrak{a}$, i.e.

$$\delta([ab]) = [1 \otimes a + a \otimes 1, \delta(b)] + [\delta(a), 1 \otimes b + b \otimes 1];$$

(ii) The map $\delta^* : \mathfrak{a}^* \otimes \mathfrak{a}^* \to \mathfrak{a}^*$ dual to δ is a Lie bracket on $\mathfrak{a}^*$.
In this case δ is called the cocommutator of $\mathfrak{a}$.



For any Lie bialgebra $\mathfrak{a}$, the vector space $\mathfrak{g} = \mathfrak{a} \oplus \mathfrak{a}^*$ has a natural structure of a Lie algebra. Namely, $\mathfrak{a}, \mathfrak{a}^*$ are Lie subalgebras in $\mathfrak{g}$ with bracket defined above, and commutator between elements of $\mathfrak{a}, \mathfrak{a}^*$ is given by

$$[a, b] = (\mathrm{ad}^* a)b - (1 \otimes b)(\delta(a)), a \in \mathfrak{a}, b \in \mathfrak{a}^*, \tag{1.1}$$

Let $\mathfrak{a}$ be a Lie algebra, and $r \in \mathfrak{a} \otimes \mathfrak{a}$. The equation

$$[r_{12}, r_{13}] + [r_{12}, r_{23}] + [r_{13}, r_{23}] = 0 \tag{1.2}$$

in $U(\mathfrak{a})^{\otimes 3}$ is called the classical Yang-Baxter equation. It is easy to check that the canonical element r satisfies this equation.

$$F(\Phi_{VWU}) J_{V \otimes W, U} \circ (J_{VW} \otimes 1) = J_{V, W \otimes U} \circ (1 \otimes J_{WU})$$

$$J = (\phi^{-1} \otimes \phi^{-1}) \Big(\Phi^{-1}_{1,2,34} (1 \otimes \Phi_{2,3,4}) s e^{h\Omega_{23}/2} (1 \otimes \Phi^{-1}_{2,3,4}) \Phi_{1,2,34} (1_+ \otimes 1_+ \otimes 1_- \otimes 1_-) \Big)$$

Theorem 1.2.

There exist "universal quantization functors"

(i) $Q : HA_{\langle \Delta - \Delta^{op}, S - S^{-1} \rangle} \to \overline{LBA_{\langle \delta \rangle}}$ such that for any Lie bialgebra $\mathfrak{a}$ over k $\widehat{Q}(\mathfrak{a}_h) = U_h(\mathfrak{a})$;

(ii) $Q^{qt} : QTHA_{\langle \Delta - \Delta^{op}, R-1, S - S^{-1} \rangle} \to \overline{QTLBA_{\langle r \rangle}}$ such that for any quasitriangular Lie bialgebra $\mathfrak{a}$ over k $\widehat{Q^{qt}}(\mathfrak{a}_h) = U_h^{qt}(\mathfrak{a})$, where $U_h^{qt}(\mathfrak{a})$ is the quasitriangular quantization defined in Section 6.1;

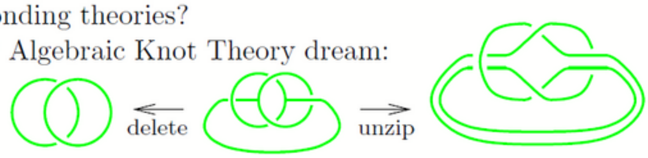
(iii) $Q^{YB} : QYBA_{\langle R-1 \rangle} \to CYBA_{\langle r \rangle}$ such that for any classical Yang-Baxter algebra (A, r) over k one has $\widehat{Q^{YB}}(A_h) = (A, R)$, where R is constructed from r as explained in Chapter 5.

So there exists a "universal formula". Does it mean anything in itself?

God doesn't create a pattern unless she means us to study it.

Why Should We Care? * Because it's there!

- A gateway into the forbidden territory of "quantum groups".
- Abstractly more pleasing: We study the things, and not just their representations.
- $\mathcal{A}^v$ is sometimes easier than $\mathcal{A}^u$: Alexander, say, arises easily from the 2D Lie algebra[4].
- Potentially, $\mathcal{A}^v$ has many more "internal quotients" than there are Lie bialgebras. What are they and what are the corresponding theories?
- My old[5] Algebraic Knot Theory dream:



(from DBN/Talks/Swissknots-1105)

Seen today at the Tel-Aviv Pride Parade, by Assaf Bar-Natan