HW7 due date postponed to Friday Nov 23!

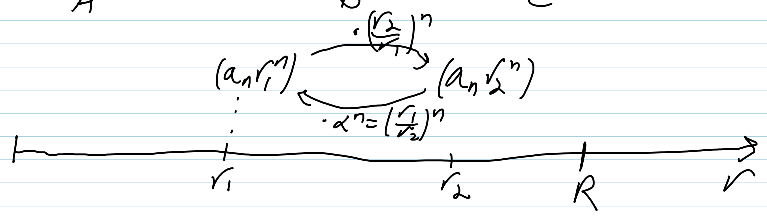
An unplanned interlude on the irrationality of π.

Little on power series.

Thm 1 Given $\sum a_n x^n$ $\exists R \in \mathbb{R}_{\geq 0} \cup \{\infty\}$, "radius of convergence", s.t. $\sum a_n x^n$ absolutely converges if $|x|<R$ & diverges if $|x|>R$. [if $|x|=R$, "it depends"].

Proof. Given a sequence b_n,

$$\underset{A}{\begin{pmatrix} b_n \text{ summable} \\ (\sum b_n \text{ converges})\end{pmatrix}} \Rightarrow \underset{B}{(b_n \to 0)} \Rightarrow \underset{C}{(b_n \text{ bounded})} \Rightarrow \overset{\text{for } \alpha<1}{\begin{pmatrix}\alpha^n b_n \\ \text{summable}\end{pmatrix}}$$



$\Rightarrow$ If A or B or C holds for some r_2, then A & B & C hold for any r_1 left for r_1 ($r_1 < r_2$).

So $R = \sup\{r : a_n r^n \underset{n\to\infty}{\longrightarrow} 0\} = \sup\{r : |a_n r^n| \text{ is bndd}\}$

Thm 2 (loose) 1. If f has a formula, it has a natural extension to $\mathbb{C}$.

2. In that case, R is the distance from 0 to the nearest point in $\mathbb{C}$ in which the formula truly fails.

Examples. 1. $\sin x = \dfrac{e^{ix}-e^{-ix}}{2i}$ 2 $\dfrac{1}{1+x^2} = 1 - x^2 + x^4 - x^6 \dots$

3. $\sum C_n x^n = \dfrac{1-\sqrt{1-4x}}{2x} = \dfrac{1-(1-4x)}{2x(1+\sqrt{1-4x})} = \dfrac{2}{1+\sqrt{1-4x}}$

.... thus C_n grows faster than 3.99^n & slower than 4.01^n.

Problem Given $y'' + p(x)y' + q(x)y = g(x)$, find a power series $y = \sum_{n=0}^{\infty} a_n x^n$ that solves this eq'n.

Do the $y''+y=0$ example. {skipped

Do the Airy example $y'' = xy$

get: $a_2 = 0$

$(n+2)(n+1)a_{n+2} = a_{n-1}$

done fine.

$y_1 = 1 + \frac{1}{2\cdot3}x^3 + \frac{1}{2\cdot3\cdot5\cdot6}x^6 + \frac{1}{2\cdot3\cdot5\cdot6\cdot8\cdot9}x^9 \dots$

$y_2 = x + \frac{1}{3\cdot4}x^4 + \frac{1}{3\cdot4\cdot6\cdot7}x^7 + \dots$

Pensieve header: Plotting the solutions of the Airy equation, $y''=xy$.

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In[1]:= Ai1 = NDSolve[y''[x] == x y[x] && y[0] == 1 && y'[0] == 0, y[x], {x, -10, 3}];
        Ai2 = NDSolve[y''[x] == x y[x] && y[0] == 0 && y'[0] == 1, y[x], {x, -10, 3}];
        Ai = Join[Ai1, Ai2]
Out[3]= {{y[x] → InterpolatingFunction[{{-10., 3.}}, <>][x]},
         {y[x] → InterpolatingFunction[{{-10., 3.}}, <>][x]}}
In[4]:= Plot[Evaluate[y[x] /. Ai], {x, -10, 3}]
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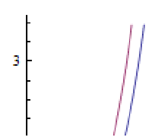

$$y_2 = x + \frac{1}{3\cdot 4}x^4 + \frac{1}{3\cdot 4\cdot 6\cdot 7}x^7 + \cdots$$

state Fuchs' Theorem: The series for $y(x)$ converges at radius at least the least of the radii for p, q, g.

Proof as in handout.

Out[4]=