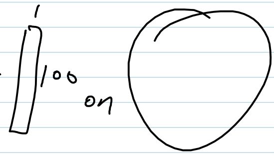
HW4 on web later today?
Riddle Along. Bread slicer?   99x   100 on



Lagrange multipliers: Find max of $f(x,y)$ on $g(x,y)$.

Silly: substitution.

Smart: Lagrange multipliers,

set $h_\lambda(x,y)=f+\lambda g$; solve

$$\nabla h_\lambda = 0$$
$$g(x,y)=0$$

E.g. find pt nearest to 0 on the curve

$$x^2+xy+y^2=1$$
$$f=x^2+y^2$$
$$g=x^2+xy+y^2-1$$
$$h_\lambda=(1+\lambda)(x^2+y^2)+\lambda xy-\lambda$$

$$\nabla h_\lambda = \begin{pmatrix}(1+\lambda)2x+\lambda y\\ (1+\lambda)2y+\lambda x\end{pmatrix}=0$$

$y=-\frac{2}{\lambda}(1+\lambda)x \qquad x=-\frac{2}{\lambda}(1+\lambda)y$

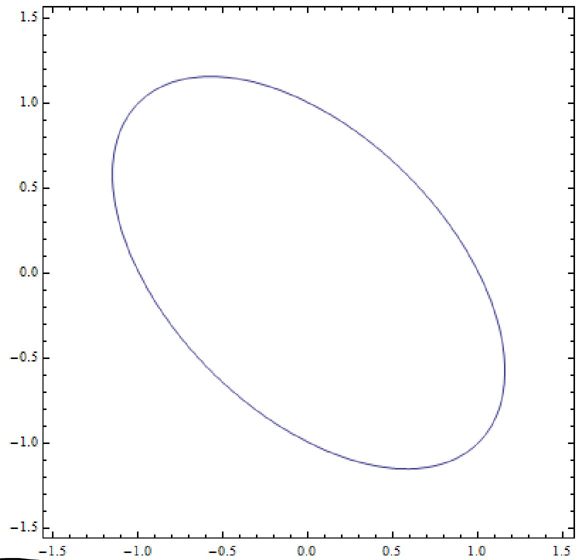
$y=\alpha x \qquad x=\alpha y \qquad \alpha=\pm 1$

$y=x$: $3x^2=1$

$y=-x$: $x^2=1$

In[1]:= ContourPlot[x^2 + x y + y^2 == 1, {x, -1.5, 1.5}, {y, -1.5, 1.5}]

Out[1]=



Example

$y(a)=0$
$y(b)=0$

$J=\int_a^b y\,dx \qquad G=\int_a^b\sqrt{1+y'^2}\,dx=l$

$J+\lambda G=\int_a^b\left(y+\lambda\sqrt{1+y'^2}\right)dx \qquad F_\lambda=y+\lambda\sqrt{1+y'^2}$

Rare case! Euler-Lagrange is simpler than its simplification!

$$0=F_y-\frac{d}{dx}F_{y'}=1-\lambda\frac{d}{dx}\frac{y'}{\sqrt{1+y'^2}} \Rightarrow \frac{\lambda y'}{\sqrt{1+y'^2}}=x-c_1$$

solve for y', get $y'=\frac{x-c_1}{\sqrt{\lambda^2-(x-c_1)^2}} \Rightarrow y-c_2=-\sqrt{\lambda^2-(x-c_1)^2}$

$$\Rightarrow (x-c_1)^2+(y-c_2)^2=\lambda^2$$

! ! !

Why do Lagrange multipliers work?

1. "gradients should be proportional"
2. An aside on directional derivatives — if $\nabla g \neq 0$, $\nabla f \not\propto \nabla g$, find v s.t. $v \cdot \nabla g = 0$ yet $v \cdot \nabla f \neq 0$ & show that p isn't a stationary pt.