class photo. Tomorrow!

show & tell. $\sqrt{2}$ is irrational in two new ways:

1. $K :=$ smallest natural s.t. $K\sqrt{2} \in \mathbb{N}$. Then $K' = K(\sqrt{2}-1) < K$ yet $K'\sqrt{2} = K \cdot 2 - K\sqrt{2} \in \mathbb{N}$
2. If $\left(\frac{p}{q}\right)^2 = 2$ then $\left(\frac{2q-p}{p-q}\right)^2 = 2$, and the denominator is smaller.

Calculus (p)review. If $\psi : \mathbb{R}^n \to \mathbb{R}$ is twice differentiable, then $\frac{\partial^2 \psi}{\partial x \partial y} = \frac{\partial^2 \psi}{\partial y \partial x}$

Combinatorial analog / exercise: If $\psi : \mathbb{Z}^2 \to \mathbb{R}$ define $(\delta_x \psi)(x,y) = \psi(x+1,y) - \psi(x,y)$ and $(\delta_y \psi)(x,y) = \psi(x,y+1) - \psi(x,y)$. Then

$$\delta_x \delta_y \psi = \delta_y \delta_x \psi.$$

An equation for $\psi(x,y) = C$ is $\psi_x + \psi_y y' = 0$

How can we tell if $M + Ny' = 0$ or $M\,dx + N\,dy = 0$ is exact?

Claim This is iff $M_y = N_x$.

More precisely, if on some rectangle M, N & their derivatives exist & are continuous then

$$\exists \psi \text{ s.t. } \psi_x = M, \psi_y = N \iff M_y = N_x$$

PF $\Rightarrow$ Easy

$\Leftarrow$ Suppose $\chi_x = M$. Wish to find $\phi(y)$ s.t. $\psi = \chi + \phi$ works: $(\chi + \phi)_y = N$ i.e. $\phi_y = N - \chi_y$; possible if $0 = (N - \chi_y)_x = N_x - \chi_{xy} = N_x - M_y$

$M_y = \cos x + 2x e^y$ $N_x = \cos x + 2x e^y$!

$\chi = y \sin x + x^2 e^y$

תרגיל: פתור

$(y\cos x + 2xe^y) + (\sin x + x^2 e^y + 2)y' = 0$

$M_y = \cos x + 2xe^y$ $N_x = \cos x + 2xe^y$

תרגיל: פתור

$\chi = y\sin x + x^2 e^y$

$\phi = 2y$

$$(\underbrace{y\cos x + 2xe^y}_{M}) + (\underbrace{\sin x + x^2 e^y + 2}_{N})y' = 0$$

(פתרון: $\psi(x,y) = y\sin x + x^2 e^y + 2y$) אז נציב $x=0$, $y=1$ ...

6. גורמי אינטגרציה:

לפעמים $M + Ny' = 0$ הופכת מדויקת אם מכפילים ב-μ ...

התנאי על μ:

$$M\mu_y - N\mu_x + (M_y - N_x)\mu = 0$$

זו מד"ח. יכולים להיות הרבה פתרונות, וכולם יהיו טובים. מאוד קשה למצוא)

אולם, אם יש הנחות:

μ תלוי רק ב-x:

$$\frac{\mu_x}{\mu} = \frac{M_y - N_x}{N}$$

ניתן לפתור אם RHS תלוי רק ב-y.

μ depends only on y:

$$\frac{M\mu_y}{\mu} = N_x - M_y$$

$$\frac{\mu_y}{\mu} = \frac{N_x - M_y}{M}$$

depends only on y

דוגמא: ~~פתור~~ פתור

$$(\underbrace{3xy + y^2}_{M}) + (\underbrace{x^2 + xy}_{N})y' = 0$$

done line

$$\frac{M_y - N_x}{N} = \frac{3x + 2y - 2x - y}{x^2 + xy} = \frac{x+y}{x(x+y)} = \frac{1}{x}$$

$$\frac{\mu_x}{\mu} = \frac{1}{x} \Rightarrow \mu = x$$

$$(3x^2y + xy^2)\,dx + (x^3 + x^2y)\,dy = 0$$

$$\psi = x^3y + \tfrac{1}{2}x^2y^2 + \phi(y)$$

$$\psi_y = x^3 + x^2y + \phi' \qquad \psi = x^3y + \tfrac{1}{2}x^2y^2 = C$$