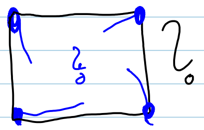
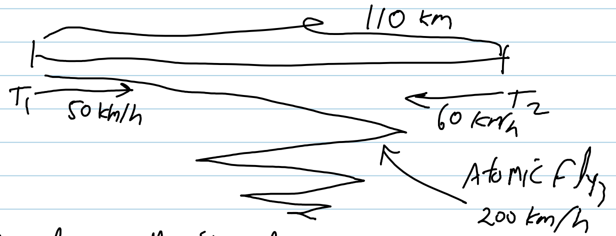
* All who requested a wiki account, received it.
* If it's not linked, it doesn't exist.

Read Along. BDP chapter 1 & 2.1–2.2, 2.4–2.6

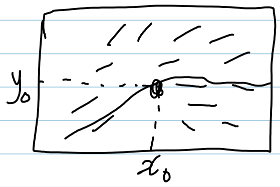
Riddle Along.





Wed: A better fly bouncing problem | How long will it fly before crashing?

$$y' = f(x,y)$$
$$y(x_0) = y_0$$



Separable equations: $m(x) + n(y)\frac{dy}{dx} = 0$ | $y' = f(x)\cdot g(y)$

4. הפרדת משתנים:

$m(x) + n(y)y' = 0$

דוגמה: $y' = \frac{3x^2+4x+2}{2(y-1)}$, $y(0) = -1$

השיטה הקלה להצדקה:

נגדיר פונק׳ $M(x)$ ו־$N(y)$ כך ש $M_x = m$, $N_y = n$

$$M_x(x) + N_y(y)y' = 0$$
$$\frac{d}{dx}M + \frac{d}{dx}N(y) = 0$$
$$\frac{d}{dx}\left(M(x) + N(y)\right) = 0$$
$$M(x) + N(y) = C$$

השיטה הנוחה לזכירה:

$$m(x)dx + n(y)dy = 0$$
$$\int m(x)\,dx + \int n(y)\,dy = C$$

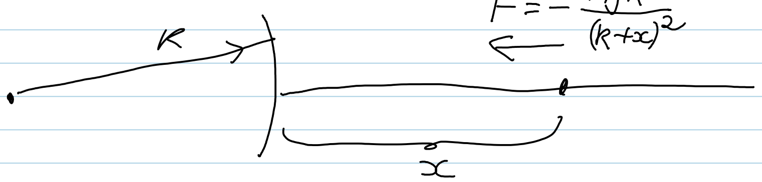
דוגמה: $2(y-1)dy = (3x^2+4x+2)dx$

$$y^2 - 2y = x^3 + 2x^2 + 2x + C \qquad C = 3$$
$$y = 1 \pm \sqrt{x^3 + 2x^2 + 2x + 4}$$
$(-)$

Example: The brachistochrone. $y' = \sqrt{\frac{d-y}{y}}$

Example: Escape velocities

$F = -\frac{mgR^2}{(R+x)^2}$ ← scales right & correct at $x = 0$.



So

$$m\frac{dv}{dt} = -\frac{mgR^2}{(R+x)^2}$$

$$\frac{dv}{dt} = \frac{dv}{dx}\frac{dx}{dt} = v\cdot\frac{dv}{dx} \Rightarrow v\frac{dv}{dx} = -\frac{gR^2}{(R+x)^2}$$

done

$$\frac{dv}{dt} = \frac{dv}{dx}\frac{dx}{dt} = v \cdot \frac{dv}{dx} \Rightarrow v\frac{dv}{dx} = -\frac{gR^2}{(R+x)^2}$$

done line

$$\frac{v^2}{2} = \frac{gR^2}{R+x} + C \qquad v(0) = v_0 \Rightarrow C = \frac{v_0^2}{2} - gR$$

So $\frac{v^2}{2} = \frac{gR^2}{R+x} - gR + \frac{v_0^2}{2}$

For which v_0, $\lim_{x \to \infty} v(x) = 0$?

$$gR = \frac{v_c^2}{2} \qquad v_c = \sqrt{2gR}$$

Autonomous equations. $y' = f(y)$

"Uncovering the tracks"

Homogeneous equations.

משוואות הומוגניות: משוואות שהנגזרה: $y' = F\left(\frac{y}{x}\right)$ כגון $y' = \frac{y^2+2xy}{x^2}$

פתרון ע"י הצבה $v = \frac{y}{x}$

$$y = vx \qquad y' = v'x + v$$

$$xv' = F(v) - v \qquad x\frac{dv}{dx} = F(v) - v \qquad \frac{dx}{x} = \frac{dv}{F(v)-v}$$

Example: $y' = \frac{y^2+2xy}{x^2}$ $\qquad x\frac{dv}{dx} = v^2 + 2v - v$

$$= v(v+1)$$

$$\frac{dv}{v(v+1)} = \frac{dx}{x} \qquad \log|x| + C = \int \frac{dv}{v(v+1)} = \int \left(\frac{-1}{v+1} + \frac{1}{v}\right) dv$$

$$= -\log|v+1| + \log|v| = \log\left|\frac{v}{v+1}\right|$$

$$cx = \frac{v}{v+1} \qquad (v+1)cx = v \qquad v(cx-1) = -cx$$

$$v = \frac{-cx}{cx-1} \qquad y = \frac{-cx^2}{cx-1}$$