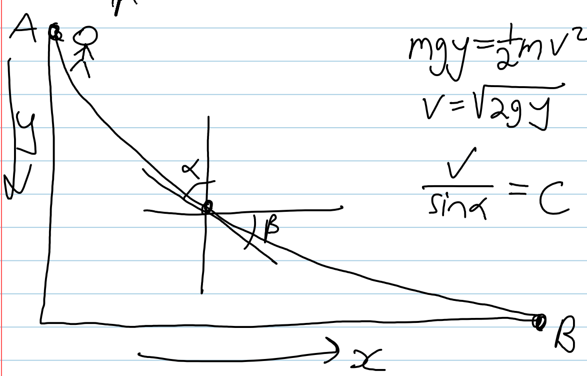
267ODEs-120911, Hour 2: More on the brachistochrone, the "about" handout

September-10-12
1:40 PM

Example. The Brachistochrone:



$mgy = \frac{1}{2} m v^2$

$v = \sqrt{2gy}$

$\frac{v}{\sin\alpha} = C$

So $\sqrt{2gy} \cdot \sqrt{1+y'^2} = C$ or better

start line

$y(0) = 0$
$y(B) = A$

$y(1+y'^2) = d$

How solve? That's why we're here!

```
In[1]:= DSolve[y[x] Sqrt[1 + (y'[x])^2] == C2, y[x], x]
Out[1]= {{y[x] -> -Sqrt[-x^2 - 2 x C[1] - C[1]^2 + C2^2]},
         {y[x] -> Sqrt[-x^2 - 2 x C[1] - C[1]^2 + C2^2]},
         {y[x] -> -Sqrt[-x^2 + 2 x C[1] - C[1]^2 + C2^2]},
         {y[x] -> Sqrt[-x^2 + 2 x C[1] - C[1]^2 + C2^2]}}
```

Better, $1 + y'^2 = d/y$

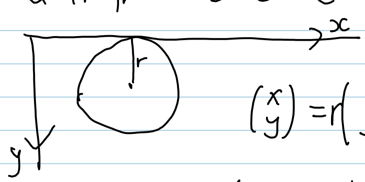
$y' = \sqrt{\frac{d-y}{y}}$

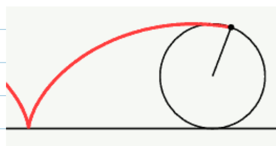
$\frac{dy}{dx} = \sqrt{\frac{d-y}{y}}$

$\sqrt{\frac{y}{d-y}}\, dy = dx \Rightarrow \int \sqrt{\frac{y}{d-y}}\, dy = x + C_1$

```
In[5]:= Integrate[Sqrt[y/(d - y)], y] // FullSimplify
Out[5]= (Sqrt[y/(-y + c2)] (y^(3/2) + c2 (-Sqrt[y] + ArcTan[Sqrt[y]/Sqrt[-y + c2]] Sqrt[-y + c2]))) / Sqrt[y]
```

claim: This is a "cycloid".





http://en.wikipedia.org/wiki/Cycloid

$\binom{x}{y} = r\binom{-\sin t}{-\cos t} + \binom{tr}{r}$

$\rightsquigarrow r\binom{t - \sin t}{1 - \cos t}$ $\quad x(t) = r(t - \sin t)$, $\quad y(t) = r(1 - \cos t)$

$\frac{dy}{dx} = \frac{dy}{dt} \cdot \left(\frac{dx}{dt}\right)^{-1} = \frac{\sin t}{1 - \cos t}$

Is there d for which $y' = \sqrt{\frac{d-y}{y}}$?

Works for $d = 2r$:

$\left(\frac{S}{1-C}\right)^2 \stackrel{?}{=} \frac{2r - r(1-C)}{r(1-C)} = \left(\frac{1+C}{1-C}\right)$ $\quad \frac{1-C^2}{(1-C)^2} \stackrel{?}{=} \frac{1+C}{1-C}$ $\quad \frac{(1-C)(1+C)}{(1-C)^2} = \frac{1+C}{1-C}$ ✓

Go over video page & "About" handout

done

Go over video page & "About" handout

done
time

Words: "Differential Equation"

"Ordinary"

"Partial"

"Order"

Linear → homogeneous

Linear → non-homogeneous