

Pensieve header: Finding a power series solution of an arbitrary ODE of the form  $y' = f(x,y)$ .

```
In[1]:= PowerSeriesSolve[f_, x0_, y0_, n_] := (
  ϕ[0] = y0;
  Do[
    ϕ[k] = y0 + ∫x0x (Normal[Series[f /. y → ϕ[k-1], {x, x0, k-1}]] /. x → t) dt,
    {k, 1, n}
  ];
  ϕ[n]
);
```

```
In[5]:= PowerSeriesSolve[Sqrt[1+y^2], 0, 0, 5]
```

```
Out[5]= x +  $\frac{x^3}{6}$  +  $\frac{x^5}{120}$ 
```

```
PowerSeriesSolve2[f_, x0_, y0_, n_] := Module[{ϕ = y0},
  Do[
    ϕ +=  $\frac{D[f /. y \rightarrow \phi, \{x, k-1\}] /. x \rightarrow x0}{k!} x^k$ ,
    {k, 1, n}
  ];
  ϕ
];
```

```
In[6]:= PowerSeriesSolve2[Sqrt[1+y^2], 0, 0, 10]
```

```
Out[6]= x +  $\frac{x^3}{6}$  +  $\frac{x^5}{120}$  +  $\frac{x^7}{5040}$  +  $\frac{x^9}{362880}$ 
```

```
In[7]:= Series[Sinh[x], {x, 0, 10}]
```

```
Out[7]= x +  $\frac{x^3}{6}$  +  $\frac{x^5}{120}$  +  $\frac{x^7}{5040}$  +  $\frac{x^9}{362880}$  + O[x]11
```