Pensieve header: A comparison of Euler, Improved Euler, and Runge-Kutta on $y'=-y$.

```
Euler[f_, x0_, y0_, x_, n_, p_] := Module[
  {h = N[(x - x0) / n, p], xj = N[x0, p], yj = N[y0, p], k1},
  Do[
   k1 = f[xj, yj];
   xj = xj + h;
   yj = yj + h k1,
   {n}
  ];
  yj
 ]
```

```
f1[x_, y_] := -y;
{Euler[f1, 0, 1, 1, #, 10] & /@ {1, 10, 10^2, 10^3, 10^4, 10^5}, N[1 / E, 10]} // Timing
```

```
{0.437, {{0. × 10^-10, 0.348678440, 0.366032341,
    0.367695425, 0.367861046, 0.367877602}, 0.3678794412}}
```

```
ImprovedEuler[f_, x0_, y0_, x_, n_, p_] := Module[
  {h = N[(x - x0) / n, p], xj = N[x0, p], yj = N[y0, p], k1, k2},
  Do[
   k1 = f[xj, yj];
   k2 = f[xj + h, yj + h k1];
   xj = xj + h;
   yj = yj + h (k1 + k2) / 2,
   {n}
  ];
  yj
 ]
```

```
ImprovedEuler[f1, 0, 1, 1, #, 10] & /@ {1, 10, 10^2, 10^3, 10^4, 10^5} // Timing
```

```
{0.905, {0.500000000, 0.368540985, 0.367885619, 0.367879503, 0.367879442, 0.367879441}}
```

```
RungeKutta[f_, x0_, y0_, x_, n_, p_] := Module[
  {h = N[(x - x0) / n, p], xj = N[x0, p], yj = N[y0, p], k1, k2, k3, k4},
  Do[
   k1 = f[xj, yj];
   k2 = f[xj + h / 2, yj + h k1 / 2];
   k3 = f[xj + h / 2, yj + h k2 / 2];
   k4 = f[xj + h, yj + h k3];
   xj = xj + h;
   yj = yj + h (k1 + 2 k2 + 2 k3 + k4) / 6,
   {n}
  ];
  yj
 ]
```

```
{RungeKutta[f1, 0, 1, 1, #, 20] & /@ {1, 10, 10^2, 10^3, 10^4, 10^5}, N[1 / E, 20]} //
 Timing
```

```
{2.215, {{0.37500000000000000000,
    0.3678797744124984334, 0.3678794412023555116, 0.3678794411714453898,
    0.3678794411714423219, 0.3678794411714423216}, 0.36787944117144232160}}
```