Do not turn this page until instructed.

Math 267 Advanced Ordinary Differential Equations

Sample Term Test

University of Toronto, October 19, 2012

Solve the 3 of the 4 problems on the other side of this page.
Each problem is worth 34 points.
You have fifty minutes to write this test.

Notes.

- No outside material other than stationary and a basic calculator (not capable of displaying text) is allowed.

- **Neatness counts! Language counts!** The *ideal* written solution to a problem looks like a page from a textbook; neat and clean and made of complete and grammatical sentences. Definitely phrases like "there exists" or "for every" cannot be skipped. Lectures are mostly made of spoken words, and so the blackboard part of proofs and calculations given during lectures often omits or shortens key phrases. The ideal written solution to a problem does not do that.

Good Luck!

Solve 3 of the following 4 problems. Each problem is worth 34 points. You have fifty minutes. **Neatness counts! Language counts!**

Problem 1. Solve the following two differential equations:

1. Linear non-homo similar to HW.

2. Exact similar to HW.

Problem 2.

1. State precisely (without proof) the theorem about existance and uniqueness of solutions for *systems* of ordinary differential equations of order 1.

2. Show how the above theorem implies an existance and uniqueness theorem for ordinary differential equations of arbitrary order.

Problem 3. CoV similar to HW.

Problem 4.

1. State what is the "improved Euler method" for solving the differential equation $\phi' = f(x, \phi)$ with initial condition $\phi(x_0) = y_0$ using step size h.

2. Compute the single-step approximation for $\phi(1)$, given that ϕ satisfies $\phi' = -\phi$ and $\phi(0) = 1$, using the improved Euler method.

Warning: The real exam will be similar to this sample, to my opinion. Your opinion may be significantly different.