240AlgebraI-121129, Hour 34: Deriving the basic row properties of dets, matrix powers

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Goal: (2nd goal: matrix powers)

$|E^1_{i,j}A| = -|A| \quad |E^2_{i,c}A| = c|A| \quad |E^3_{i,j,c}A| = |A|$

$|(a_{11})| := a_{11} \quad \left|\begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & & \\ a_{n1} & \cdots & a_{nn}\end{pmatrix}\right| := \sum_{j=1}^{n}(-1)^{1+j}a_{1j}|A_{\hat{1}\hat{j}}|$

1. Linear in the first row.
2. Multilinear in the rows.
3. Vanishes if the first two rows are equal.
4. Vanishes if two adjacent rows are equal.
5. Switches sign if two adjacent rows are interchanged.
6. Switches sign whenever two rows are interchanged.
7. $E^2_{i,c}$ & $E^2_{i,j,c}$ behavior.

Problem. For any $A \in M_{n\times n}(F)$, compute A^p.

Example: $\begin{pmatrix} 4 & 3 \\ -6 & -5\end{pmatrix}^{15} = ?$ [here $C^{-1}AC = D$, w/ $D = \begin{pmatrix} 1 & 0 \\ 0 & -2\end{pmatrix}, C = \begin{pmatrix} 1 & -1 \\ -1 & 2\end{pmatrix}$]

Brilliant idea: If $A = CDC^{-1}$ for some C & a diagonal D, all is easy.

Painfully, we'll skip the context in which this is natural.

Anyway, it works.....

But how do we find C & D?

$D = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2\end{pmatrix} \quad C = \left(v_1 \middle| v_2\right) \quad AC = CD \quad (*)$

Evaluate both sides of (*) on e_1, e_2, get

$$Av_1 = \lambda_1 v_1 \quad , \quad Av_2 = \lambda_2 v_2$$

done line

$$Av = \lambda v$$ "eigenvalue eigenvector"

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