HW 9 on web by midnight! Due Tue Dec 4!

Basic properties of determinants:

0. $\det(I_n) = 1$

1. $\det\begin{pmatrix} - r_j - \\ - r_i - \end{pmatrix} = -\det\begin{pmatrix} - r_i - \\ - r_j - \end{pmatrix}$

2. $\det\begin{pmatrix} \text{---} \\ - cr - \\ \text{---} \end{pmatrix} = c\det\begin{pmatrix} \text{---} \\ - r - \\ \text{---} \end{pmatrix}$

3. $\det\begin{pmatrix} - r_i - \\ \text{---} \\ - r_j + cr_i - \end{pmatrix} = \det\begin{pmatrix} - r_i - \\ \text{---} \\ - r_j - \end{pmatrix}$

Example $\det\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$

Corollary All that there is to know about determinants can be deduced from 0-3; also if $\det'$ satisfies 0-3, then $\det' = \det$.

Thm A is invertible iff $\det(A) \neq 0$

Thm If $A = E_1 \cdots E_n$ is a product of elementary matrices, then $\det A = \det(E_1)\cdot\det(E_2)\cdots\det(E_n)$

Claim For square matrices, AB invertible $\Leftrightarrow$ A & B are inv.

$\Leftarrow$ $(AB)^{-1} = B^{-1}A^{-1}$

$\Rightarrow$ $B(AB)^{-1}$ is a right inverse for A, & for square matrices, if $AC = I$ then also $CA = I$.

Thm $\det A\cdot B = \det A \cdot \det B$

Thm $\det A^T = \det A$

Thm Everything that's true for rows is also true for columns.

Skipped extras: 1. Other formulas for det. (row/col expansions, permutations)

2. A det formula for A^{-1} & Kramer's law.

1. It's in all the books.

2. I've never used it in my life.

Skipped

... recall the formula for dets & sketch the proof of the basic properties:

$$|(a_{11})| := a_{11} \qquad \left|\begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & & \\ a_{n1} & \cdots & a_{nn} \end{pmatrix}\right| := \sum_{j=1}^{n} (-1)^{1+j} a_{1j} \left|A_{\hat{1}\hat{j}}\right|$$

Then prove:

1. Linear in the first row.
2. Multilinear in the rows.
3. Vanishes if the first two rows are equal.

done line

4. Vanishes if two adjacent rows are equal.
5. Switches sign if two adjacent rows are interchanged.
6. Switches sign whenever two rows are interchanged.
7. $E^2_{i,c}$ & $E^2_{i,j,c}$ behaviour.