HW6 is on web!

Riddle along: A game: Player A writes the numbers 1-18 on the faces of three blank dice, to her liking. Player B takes one of the 3 dice. Player B takes one of the remaining two, and throws away the third. Player A and B then play 1,000 rounds of "dice war" with the dice they hold. Whom would you rather be, player A or player B?

On board: V, $\beta = (u_1, \dots u_n)$ an ordered basis

Given $x \in V$, write $x = \sum a_i u_i$

$$[x]_\beta = \begin{pmatrix} a_1 \\ \vdots \\ a_n \end{pmatrix}$$

"coords of x rel β"

<u>Thm.</u> Given V w/ basis $\beta = (v_1 \dots v_n)$
and W w/ basis $\gamma = (w_1 \dots w_m)$

We have an isomorphism

Abstract, general, coord-free — merely numbers, choice-dependent, easy to work with

$$\mathcal{L}(V, W) \longrightarrow M_{m \times n}(F)$$

$$T \longrightarrow [T]_\beta^\gamma = A$$

$$A = \begin{pmatrix} a_{11} & & & a_{1n} \\ [Tv_1]_\gamma & [Tv_2]_\gamma & \cdots & [Tv_n]_\gamma \\ a_{m1} & & & a_{mn} \end{pmatrix} \iff T v_j = \sum_{i=1}^{m} a_{kj} w_k$$

<u>Examples</u> 0. 0 1. I

2. $D: P_3(\mathbb{R}) \longrightarrow P_2(\mathbb{R})$ differentiation
3. $T_\alpha : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$
4. $A: F^n \longrightarrow F^m$

Complete the proof that this is a vector space iso.

$$\begin{tikzcd} U \arrow[r, "S"', "B"] \arrow[rr, bend right, "T \circ S", "C"'] & V \arrow[r, "T"', "A"] & W \end{tikzcd}$$

basis: α β γ
dim p n m

If you know A and B, can you find C?
⋮
derive C, then:

done line

__Definition__ $A \in M_{m\times n}$, $B \in M_{n\times p}$ $A \cdot B \in M_{m\times p}$ by $(A\cdot B)_{ik} = \sum_{j=1} A_{ij} B_{jk}$

__Example__ $\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ -1 & 1 \end{pmatrix} = \ldots$

__Thm__ $[T \circ S]_\alpha^\gamma = [T]_\beta^\gamma \cdot [S]_\alpha^\beta$

__Example__ $T_\beta \circ T_\alpha = T_{\alpha+\beta}$ for rotations.

The good and the bad about "matrix algebra":

Good	Bad
1. $A+B=B+A$, $(A+B)+C = A+(B+C)$ (basically, all works for addition)	1. Addition is defined only for matrices of same dims.
2. $A(B\cdot C)=(A\cdot B)C$ $\exists I$ s.t. $A\cdot I=A$, $IA=A$	2. mult. is defined only if "mid" dimension matches, & produces an output of yet other dims.
3. If $A\cdot A^{-1}=I$, then $A^{-1}A=I$	3. A^{-1} may not exist even if $A\neq 0$.
4. $(A+B)C = AC+BC$ $A(B+C)=AB+AC$	4. Generally, $A\cdot B\neq B\cdot A$, even when both make sense.

Next goals: 1. Compute rank T/A.

2. Compute A^{-1} (when possible)

3. Solve systems of linear eqn's.