

HW6 is on web!

Riddle along: A game: Player A writes the numbers 1-18 on the faces of three blank dice, to her liking. Player B takes one of the 3 dice. Player B takes one of the remaining two, and throws away the third. Player A and B then play 1,000 rounds of "dice war" with the dice they hold. Whom would you rather be, player A or player B?

On board: V , $\beta = (u_1, \dots, u_n)$ an ordered basis
 Given $x \in V$, write $x = \sum a_i u_i$

$$[x]_{\beta} = \begin{pmatrix} a_1 \\ \vdots \\ a_n \end{pmatrix}$$

"coords of x rel β "

Thm. Given V w/ basis $\beta = (v_1, \dots, v_n)$
 and W w/ basis $\gamma = (w_1, \dots, w_m)$
 We have an isomorphism

Abstract, general, coord-free matrix numbers, choice-dependent, easy to work with

$$\mathcal{L}(V, W) \longrightarrow M_{m \times n}(F)$$

$$T \longrightarrow [T]_{\beta}^{\gamma} = A$$

$$A = \begin{pmatrix} a_{11} & \dots & a_{1n} \\ [Tv_1]_{\gamma} & \dots & [Tv_n]_{\gamma} \\ \vdots & & \vdots \\ a_{m1} & \dots & a_{mn} \end{pmatrix} \iff Tv_j = \sum_{i=1}^m a_{ij} w_i$$

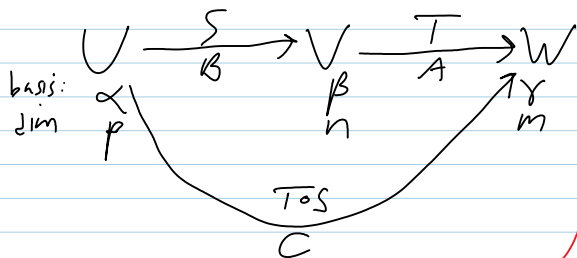
Examples 0. 0 1. 1

2. $D: P_3(\mathbb{R}) \rightarrow P_2(\mathbb{R})$ differentiation

3. $T_{\alpha}: \mathbb{R}^2 \rightarrow \mathbb{R}^2$

4. $A: F^n \rightarrow F^m$

Complete the proof that this is a vector space iso.



If you know A and B , can you find C ?

derive C , then:

done line

Definition $A \in M_{m \times n}$, $B \in M_{n \times p}$ $A \cdot B \in M_{m \times p}$ by $(A \cdot B)_{ik} = \sum_{j=1}^n A_{ij} B_{jk}$

Example $\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ -1 & 1 \end{pmatrix} = \dots$

Thm $[T \circ S]_{\alpha}^{\gamma} = [T]_{\beta}^{\gamma} \cdot [S]_{\alpha}^{\beta}$

Example $T_{\beta} \circ T_{\alpha} = T_{\alpha + \beta}$ For rotations.

The good and the bad about "matrix algebra":

Good	Bad
1. $A+B=B+A$, $(A+B)+C=A+(B+C)$ (basically, all works for addition)	1. Addition is defined only for matrices of same dims.
2. $A(B \cdot C) = (A \cdot B)C$ $\exists I$ s.t. $A \cdot I = A$, $I \cdot A = A$	2. mult. is defined only if "in" dimension matches & produces an output of yet other dims.
3. $\exists I$ s.t. $A \cdot A^{-1} = I$, then $A^{-1} \cdot A = I$	3. A^{-1} may not exist even if $A \neq 0$.
4. $(A+B)C = AC + BC$ $A(B+C) = AB + AC$	4. Generally, $A \cdot B \neq B \cdot A$, even when both make sense.

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- Next goals:
1. Compute rank T/A .
 2. Compute A^{-1} (when possible)
 3. Solve systems of linear eqn's.