240Algebral-121018, Hour 18: Linear Transformations

October-18-12
9:32 AM

Read Along: Sections 2.1-2.4

Riddle Along: 1 2 3 4 5 6 7 8 9

Two players alternate drawing cards from the above deck. The first player to have 3 cards that add up to 15, wins. Would you like to be the first to move or the second? (More on today's web, including a video link).

Term Test: Thu Oct 25 3-5PM, Examination Facility room EX 200 (east side of McCaul St., near College).

Following 09-240 hour 16, taught by Yael Karshon:

- "T:V->W is linear"
- Preserving 0.
- Claim on cx+y.
- Claim on differences and many-element sums.
- Example: R^2->R^2 by explicit formula.
- Example: Differentiation, multiplication by x.
- Example: Matrices and linear transformations on F^n.
- Example: Rotation (+ explicit formula).
- Added 2012: \calL(V,W) is a vector space.
- Composition of linear trans is a linear trans.
- Composition is non-commutative. Example: differentiation and muliplication by x.
- For a l.t., arbitrary values on a basis.
- "Isomorphism".

Let V & W be v.s. over the same field F.

A function $T: V \to W$ is a lin trans iff 1. $T(0)=0$ 2. $T(x+y)=T(x)+T(y)$ 3. $T(cx)=cT(x)$

Annoying Claim: T is linear iff $\forall c, x, y$ $T(cx+y) = cT(x)+T(y)$.

Silly claims: $T(x-y)=T(x)-T(y)$,

$$T(\sum a_i x_i) = \sum a_i T(x_i)$$

Examples 1. $T\left(\begin{pmatrix} a_1 \\ a_2 \end{pmatrix}\right) = \begin{pmatrix} 3a_1 - a_2 \\ 2a_1 + a_2 \end{pmatrix}$ $T: \mathbb{R}^2 \to \mathbb{R}^2$

2. $T(P) = P'$ $T: P_n(\mathbb{R}) \to P_{n-1}(\mathbb{R})$

Thu oct 19 hour 18.

3. $T_\theta(x)$ = counter clockwise by θ.

Theorem If $(\alpha_i)_{i=1}^n$ is a basis of V and $w_i \in W$,
There is a unique linear $T: V \to W$ s.t. ... (con't)

Cont

$T(\alpha_i) = w_i$

Pf

~~Thm~~ Def V & W are "isomorphic" if . . .

Thm Any two vector spaces of dim n are isomorphic.
In particular, all are isomorphic to F^n.

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null space / kernel
range / image
Thm These are linear spaces

~~Thm~~ Def nullity
rank

Thm nullity + rank = dim