240Algebral-121016, Hours 16-17: More bases, Lagrange Interpolation

October-14-12
1:03 PM

Read Along
Riddle Along.

Term test on Thu oct 25 at "UofT Examination Facility"

My expectations: 1. complete Mastery of the material.

2. Yes, meaning every single proof.

Find old TT's on previous years web sites.

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3. Assume $\dim V = n$. Then

a. If G generates V, $|G| \geq n$ & a subset of G is a basis.
if also $|G| = n$, then G itself is a basis.

b. If L is linearly indep in V, then $|L| \leq n$;
if also $|L| = n$, L is a basis.
if also $|L| < n$, L can be extended to a basis.

start line

4. If V is finite-dimensional and $W \subset V$ is a subspace, then W is f.d. and $\dim W \leq \dim V$. (done minus "W is f.d.")

If also $\dim W = \dim V$, then $W = V$.

If also $\dim W < \dim V$, then any basis of W can be extended to a basis of V. (not done)

Fishy Thm: Every v.s. has a basis.
[Including $\mathbb{R}/\mathbb{Q}$]

The Lagrange interpolation formula:

Let x_i be distinct pts in $\mathbb{R}/F$ $\quad i = 1, \ldots, n+1$

Let y_i be any pts in $\mathbb{R}/F$.

Q Can you find a polynomial $P \in P_n(\mathbb{R})$ s.t. $P(x_i) = y_i$?

Is it unique?

Who cares? * Scientists.

* Computer drawing programs.

Solution Let $\tilde{P}_i(x) = \prod_{j \neq i} (x - x_j)$

Then $P_i(x_j) = \begin{cases} 0 & j \neq i \\ \neq 0 & i = j \end{cases}$

set $P_i(x) = \tilde{P}_i(x) / \tilde{P}_i(x_i) = \ldots$

Then * $P(x) := \sum y_i P_i(x)$ satisfies $P(x_i) = y_i$

Follow through w/ example.

$P(0) = 5$
$P(1) = 2$
$P(3) = 2$

$P_1 = \frac{(x-1)(x-3)}{3} = \frac{1}{3}(x^2 - 4x + 3)$

$P_2 = \frac{x(x-3)}{-2} =$

$P_3 = \frac{x(x-1)}{6} = \ldots$

$p = x^2 - 4x + 5$

* $\beta = \{p_1 \dots p_{n+1}\}$ is lin. indep.
* $\Rightarrow \beta$ is a basis
* Every $f \in P_n(\mathbb{R})$ can be expressed as a lin. comb. of the p_i in a unique way.
* If $q(x)$ also satisfies $q(x_i) = y_i$, then $q(x) = p(x)$.
* Therefore the solution to our problem is unique
* Aside: If $\forall i \; p(x_i) = 0$, then $p = 0$

(so a non-zero polynomial of degree n has at most n roots.)

done line

2009 hour 16:

Taught by Yael Karshon:

- "T:V->W is linear"
- Preserving 0.
- Claim on cx+y.
- Claim on differences and many-element sums.
- Example: R^2->R^2 by explicit formula.
- Example: Differentiation.
- Example: Rotation (+ explicit formula).
- Composition of linear trans is a linear trans.
- For a l.t., arbitrary values on a basis.
- "Isomorphism".