

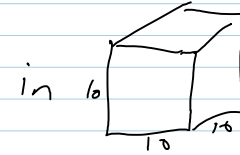
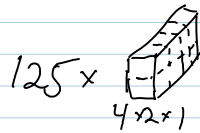
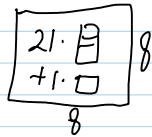
September-26-12
2:00 PM

Read Along. 1.4-1.6.

Speed up!

HW3 on Web!

Riddle Along.



Def $W \subset V$ is a "subspace" if it is a vector space with the operations it inherits from V .

Thm A non-empty $W \subset V$ is a subspace iff it is "closed under addition and under multiplication by a scalar".

Examples 1. $\{A \in M_{n \times n}(F) : A^t = A\}$

2. $\{A \in M_{n \times n}(F) : \text{tr } A = 0\}$

3. If W_1 & W_2 are subspace of V ,

then so is $W_1 \cap W_2$ (What about unions?)

Goal: Every v.s. has a "basis". So while we don't have to use coordinates, we can.

Def u is a l.c. of u_1, \dots, u_n if $\exists \alpha_i \in F$ s.t. $u = \sum \alpha_i u_i$

Examples 1. Vitamins as in the handout

2. In $P_3(\mathbb{R})$, $2x^3 - 2x^2 + 12x - 6$ is a l.c. of $x^3 - 2x^2 - 5x - 3$

and $3x^3 - 5x^2 - 4x - 9$

but $3x^3 - 2x^2 + 7x + 8$ isn't.



Thm If $\{u_i\} \subset V$ then $W = \text{span}(u_i) := \{ \text{all l.c. of the } u_i \}$ is a subspace of V .

Def $S \subset V$ "generates" or "spans" V . (First requirement from "a basis")

Examples In $V = M_{2 \times 2}(\mathbb{R})$ $M_1 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$ $M_2 = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$ $M_3 = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$

$M_4 = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$ $N_1 = \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}$... $N_4 = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}$, Then

$M_1 \dots M_4$ & $N_1 \dots N_4$ generate V , but

$M_1 \dots M_3$ & $N_1 \dots N_3$ do not.

Aside: If $S_1 \subset \text{span}(S_2)$ then $\text{span}(S_1) \subset \text{span}(S_2)$

$(N_1, N_2, N_3 \text{ are in } \{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} : a+b+c=2d \})$

$(N_1, N_2, N_3 \text{ are in } \left\{ \begin{pmatrix} a \\ b \\ c \end{pmatrix} : a+b+c=2 \right\})$
 $\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$

Def A subset $S \subset V$ is "lin. dep" if it is "wasteful".

done
line

I.e., if $\exists a_i \in \mathbb{F}$ not all 0 & $\sum a_i u_i = 0$.

Otherwise, it is "lin. indep."

Examples $\{u_i\}$ ✓, $\left\{ \begin{pmatrix} 1 \\ 1 \\ 3 \end{pmatrix}, \begin{pmatrix} 4 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 7 \\ 1 \\ 1 \end{pmatrix} \right\}$

Comments 1. \emptyset is lin. indep.

2. $\{u\}$ is lin indep iff $u \neq 0$.

3. Suppose $S_1 \subset S_2 \subset V$. Then

a. If S_1 is dep, so is S_2

b. If S_2 is indep, so is S_1

4. If S' is lin indep in V and $v \in V \setminus S'$, then

$S' \cup \{v\}$ is lin. dep. iff $v \in \text{span}(S')$.