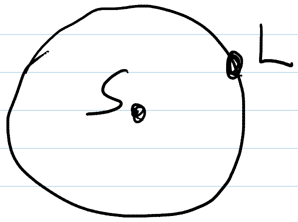
Id yourself in class photo!
Filenames must begin w/ 12-240!
Read Along. 1.1–1.4.
Riddle Along.



$V_L = 4V_S$
Can the student escape?

VS1. $x+y=y+x$ VS2: Assoc.
VS3. 0 VS4: $-$
VS5: $1\cdot x = x$ VS6 $a(bx)=(ab)x$
VS7 $a(x+y)$ VS8 $(a+b)x$

Thm
1. Cancellation law: additive, $2\times$ multiplicative.
2. 0_V is unique
3. negatives are unique.
5. $0\cdot x = 0$ 6. $a\cdot 0 = 0$
7. $(-a)x = -(ax) = a(-x)$
8. $cv=0 \Rightarrow c=0 \vee v=0$

stated, not yet proven.

start line

Def $W\subset V$ is a "subspace" if it is a vector space with the operations it inherits from V.

Thm $W\subset V$ is a subspace iff it is "closed under addition and under multiplication by a scalar" & non-empty. } not proven yet

done line

Examples.
1. $\{A\in M_{n\times n}(F) : A^t = A\}$
2. $\{A\in M_{n\times n}(F) : tr\, A = 0\}$
3. If W_1 & W_2 are subspace of V, the so is $W_1\cap W_2$ (what about unions?)

Thm So is $W_1 \cap W_2$ (What about unions?)

Goal: Every v.s. has a "basis". So while we don't <u>have</u> to use coordinates, we can.

<u>Def</u> u is a l.c. of $u_1 \ldots u_n$ if $\exists a_i \in F$ s.t. $u = \sum a_i u_i$

<u>Examples</u> 1. Vitamins as in the handout

2. In $P_3(\mathbb{R})$, $2x^3-2x^2+12x-6$ is a l.c. of x^3-2x^2-5x-3 and $3x^3-5x^2-4x-9$

but $3x^3-2x^2+7x+8$ isn't.

<u>Thm</u> If $\{u_i\} \subset V$ then $W = \text{span}(u_i) := \{\text{all l.c. of the } u_i\text{'s}\}$ is a subspace of V.

<u>Def</u> $S \subset V$ "generates" or "spans" V. (First requirement from "a basis")

<u>Examples</u> In $V = M_{2\times2}(\mathbb{R})$ $M_1 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$ $M_2 = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$ $M_3 = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$ $M_4 = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$ $N_1 = \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix} \ldots N_4 = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}$, Then $M_1 \ldots M_4$ & $N_1 \ldots N_4$ generate V, but $M_1 \ldots M_3$ & $N_1 \ldots N_3$ do not.

Aside: If $S_1 \subset \text{Span}(S_2)$ then $\text{Span}(S_1) \subset \text{Span}(S_2)$