

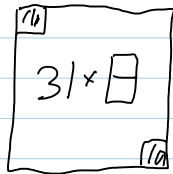
class photo at 12:55!

APUS again!

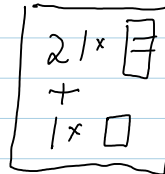
Re-iterate that wiki filenames must begin with "12-240/"; I will delete all non-compliant uploads tomorrow.

Show http://drorbn.net/index.php?title=12-240/Classnotes_for_Tuesday_September_11 as an example of a well-structured upload page.

HW2 on web
Riddle Along.



?



?

Def Let F be a field. A v.s. over F is a set V , with a special element $0 \in V$, a binary $+$: $V \times V \rightarrow V$ and a binary \cdot : $F \times V \rightarrow V$, s.t.

VS1. $x+y = y+x$

VS2: Assoc.

start
line

VS3. 0

VS4: $-$

VS5: $1 \cdot x = x$

VS6 $a(bx) = (ab)x$

VS7 $a(x+y)$

VS8 $(a+b)x$

Examples: 1. F^n

2. $M_{m \times n}(F)$

3. $\mathcal{F}(S, F)$ S a set

4. Polynomials $P_n(F)$

5. \mathbb{C}/\mathbb{R} \mathbb{R}/\mathbb{Q} "Galois theory"

Thm 1. Cancellation law: additive, $2 \times$ multiplicative.

2. 0_V is unique

3. negatives are unique.

5. $0 \cdot x = 0$ 6. $a \cdot 0 = 0$

7 $(-a)x = -(ax) = a(-x)$

stated,
not yet
proven.

$$f. CV=0 \Rightarrow C=0 \vee V=0$$

Goal: Every v.s. has a "basis". So while we don't have to use coordinates, we can.

Def $W \subset V$ is a "subspace" if it is a vector space with the operations it inherits from V .

Thm $W \subset V$ is a subspace iff it is "closed under addition and under multiplication by a scalar".

- Examples
- $\{A \in M_{n \times n}(F) : A^t = A\}$
 - $\{A \in M_{n \times n}(F) : \text{tr } A = 0\}$
 - If W_1 & W_2 are subspace of V ,
Then so is $W_1 \cap W_2$ (What about unions?)

Def u is a l.c. of u_1, \dots, u_n if $\exists \alpha_i \in F$
s.t. $u = \sum \alpha_i u_i$

Examples 1. Vitamins as in the handout

TABLE 1.1: Vitamin Content of 100 Grams of Certain Foods

Food Name	A	B ₁	B ₂	B ₆	C	E	K	P
Apple (with skin)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Banana	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Carrot (raw)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Orange (raw)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Spinach (raw)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Whole wheat flour	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Whole wheat bread	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Whole wheat pasta	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Whole wheat cereal	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Whole wheat rice	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Whole wheat corn	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Whole wheat soybeans	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Whole wheat lentils	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Whole wheat chickpeas	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Whole wheat kidney beans	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Whole wheat black beans	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Whole wheat pinto beans	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Whole wheat navy beans	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Whole wheat lima beans	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Whole wheat garbanzo beans	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Whole wheat split peas	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Whole wheat lentil soup	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Whole wheat chickpea soup	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Whole wheat kidney bean soup	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Whole wheat black bean soup	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Whole wheat pinto bean soup	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Whole wheat navy bean soup	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Whole wheat lima bean soup	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Whole wheat garbanzo bean soup	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Whole wheat lentil soup	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Whole wheat chickpea soup	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Whole wheat kidney bean soup	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Whole wheat black bean soup	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Whole wheat pinto bean soup	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Whole wheat navy bean soup	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Whole wheat lima bean soup	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Whole wheat garbanzo bean soup	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

2. In $P_3(\mathbb{R})$, $2x^3 - 2x^2 + 12x - 6$ is
a l.c. of $x^3 - 2x^2 - 5x - 3$
and $3x^3 - 5x^2 - 4x - 9$

but $3x^3 - 2x^2 + 7x + 8$ isn't.

Thm If $\{u_i\} \subset V$ then $W = \text{span}(u_i) = \{ \text{all l.c. of the } u_i \}$
is a subspace of V .

Def $S \subset V$ "generates" or "spans" V . (First requirement from "a basis")

Examples In $V = M_{2 \times 2}(\mathbb{R})$ $M_1 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$ $M_2 = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$ $M_3 = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$
 $M_4 = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$ $N_1 = \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}$... $N_4 = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}$, Then

$M_1 \dots M_4$ & $N_1 \dots N_4$ generate V , but

$M_1 \dots M_3$ & $N_1 \dots N_3$ do not.

Aside: If
 $S_1 \subset \text{Span}(S_2)$
then
 $\text{Span}(S_1) \subset \text{Span}(S_2)$