240Algebral-120925, Hours 7-8: Vector spaces

September-20-12
7:33 PM

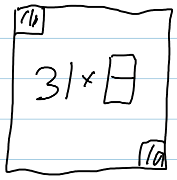
Class photo at 12:55!

APUS again!

Re-iterate that wiki filenames must begin with "12-240/"; I will delete all non-compliant uploads tomorrow.
Show http://drorbn.net/index.php?title=12-240/Classnotes_for_Tuesday_September_11 as an example of a well-structured upload page.

HW2 on web

Riddle Along.



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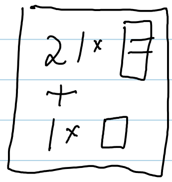

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Def Let F be a field. A v.s. over F is a set V, with a special element $0_V \in V$, a binary $+: V\times V \to V$ and a binary $\cdot: F\times V \to V$, s.t.

start line

VS1. $x+y=y+x$ VS2: Assoc.

VS3. 0 VS4: $-$

VS5: $1\cdot x = x$ VS6 $a(bx)=(ab)x$

VS7 $a(x+y)$ VS8 $(a+b)x$

Examples:
1. F^n
2. $M_{m\times n}(F)$
3. $\mathcal{F}(S,F)$ S a set.
4. Polynomials $P_n(F)$
5. $\mathbb{C}/\mathbb{R}$ $\mathbb{R}/\mathbb{Q}$ "Galois theory"

Thm 1. Cancellation law: additive, $2\times$ multiplicative.

2. 0_V is unique

3. negatives are unique.

5. $0\cdot x = 0$ 6. $a\cdot 0 = 0$

7 $(-a)x = -(ax) = a(-x)$

stated, not yet proven.

8. $cv = 0 \Rightarrow c = 0 \vee v = 0$

Goal: Every v.s. has a "basis". So while we don't have to use coordinates, we can.

Def $W \subset V$ is a "subspace" if it is a vector space with the operations it inherits from V.

Thm $W \subset V$ is a subspace iff it is "closed under addition and under multiplication by a scalar".

Examples 1. $\{A \in M_{n\times n}(F) : A^t = A\}$

2. $\{A \in M_{n\times n}(F) : \operatorname{tr} A = 0\}$

3. If W_1 & W_2 are subspaces of V, then so is $W_1 \cap W_2$ (what about unions?)

Def u is a l.c. of $u_1 \dots u_n$ if $\exists a_i \in F$ s.t. $u = \sum a_i u_i$

Examples 1. Vitamins as in the handout

2. In $P_3(\mathbb{R})$, $2x^3 - 2x^2 + 12x - 6$ is a l.c. of $x^3 - 2x^2 - 5x - 3$ and $3x^3 - 5x^2 - 4x - 9$

but $3x^3 - 2x^2 + 7x + 8$ isn't.

Thm If $\{u_i\} \subset V$ then $W = \operatorname{span}(u_i) := \{\text{all l.c. of the } u_i\text{'s}\}$ is a subspace of V.

Def $S \subset V$ "generates" or "spans" V. (First requirement from "a basis")

Examples In $V = M_{2\times2}(\mathbb{R})$ $M_1 = \begin{pmatrix}1&0\\0&0\end{pmatrix}$ $M_2 = \begin{pmatrix}0&1\\0&0\end{pmatrix}$ $M_3 = \begin{pmatrix}0&0\\1&0\end{pmatrix}$

$M_4 = \begin{pmatrix}0&0\\0&1\end{pmatrix}$ $N_1 = \begin{pmatrix}0&1\\1&1\end{pmatrix}$ ... $N_4 = \begin{pmatrix}1&1\\1&0\end{pmatrix}$, Then

$M_1 \dots M_4$ & $N_1 \dots N_4$ generate V, but $M_1 \dots M_3$ & $N_1 \dots N_3$ do not.

Aside: If $S_1 \subset \mathrm{Span}(S_2)$ then $\mathrm{Span}(S_1) \subset \mathrm{Span}(S_2)$