HW1 on web!
Read Along. Appendices A–D. (still)
Riddle Along. 3 logicians walk into a bar.
Barman: Do you all want beer?
Logician 1: I don't know
Logician 2: I don't know
Logician 3: I know.
Q: What did he know? How many had beer?

on board

Thm 1. $a+b=c+b \Rightarrow a=c$ 2. $b\neq 0,\ a\cdot b=c\cdot b \Rightarrow a=c$
3. "0 is unique" 4. "1 is unique"
5. "$-a$ is unique" 6. "a^{-1} is unique"
[So $a-b$ & a/b can be defined]
7. $-(-a)=a$, $(a^{-1})^{-1}=a$ 8. $a\cdot 0=0$

start line

9. There's no 0^{-1}
10. $(-a)\cdot b = a\cdot(-b) = -(a\cdot b)$
11. $(-a)(-b) = a\cdot b$
12. $ab=0$ iff $(a=0)\vee(b=0)$
16. $(a+b)(a-b)=a^2-b^2$

Thm $\exists!\ \chi : \mathbb{Z}\to F$ s.t. $\chi(0)=0$ $\chi(1)=1$
$\chi(m+n)=\chi(m)+\chi(n)$
$\chi(mn)=\chi(m)\cdot\chi(n)$
Def characteristic

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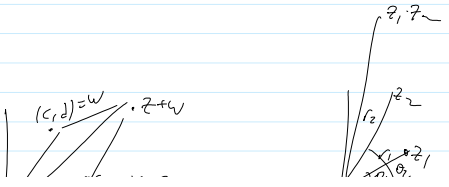
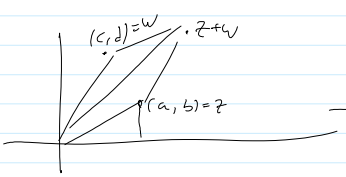
previous theme: "abstraction, generalization, definition, examples"
next: "dream, implications, formalization & proof".
Dream Add to $\mathbb{R}$ some number i so that $i^2=-1$
Implications must add $7i$, $3-7i$, $(2+3i)(3-7i)$, $(3-7i)^{-1}$, ...
Formally define $\mathbb{C}$ and verify fieldness.
Thm Our definitions indeed make a field!
show "The Complex Numbers by Computer"

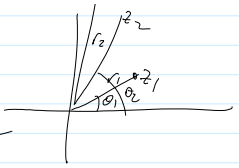
done line

Interpretation



Interpretation





Waves, AC, RLC $V = RI$, $V = I/C$

Why aren't we also adding $\sqrt{i}$?

Hour 2. V.S. and subspaces as in textbook.

Motivation: Forces can be added and multiplied by scalars

Def Let F be a field. A v.s. over F is a set V, with a special element over V, a binary $+ : V \times V \to V$ and a binary $\cdot : F \times V \to V$, s.t.

VS1. $x+y=y+x$ VS2: Assoc.

VS3. 0 VS4: $-$

VS5: $1 \cdot x = x$ VS6 $a(bx) = (ab)x$

VS7 $a(x+y)$ VS8 $(a+b)x$

Examples: 1. F^n

2. $M_{m \times n}(F)$

3. $\mathcal{F}(S,F)$ S a set

4. Polynomials

5. $\mathbb{C}/\mathbb{R}$ $\mathbb{R}/\mathbb{Q}$ "Galois theory"

Thm 1. Cancellation law: additive, 2× multiplicative.

2. 0_V is unique

3. negatives are unique.

5. $0 \cdot x = 0$ 6. $a \cdot 0 = 0$

7 $(-a)x = -(ax) = a(-x)$

8. $cV = 0 \Rightarrow c = 0 \vee V = 0$