0. Math 240 Algebra I, DROR BAR-NATAN.

1. What we'll do:

The practical side

1. Solve $5x_1 - 2x_2 + x_3 = 9$

$\begin{pmatrix} how? \\ when? \\ one/many? \end{pmatrix}$ $-x_1 + x_2 - x_3 = -2$

$2x_1 + 9x_2 - 3x_3 = -4$

this describes the small-scale behaviour of almost everything that has a mathematical description?

2. $\hookrightarrow \begin{pmatrix} 5 & -2 & 1 \\ -1 & 1 & -1 \\ 2 & 9 & -3 \end{pmatrix} = A$ "matrices" can add, multiply, take powers: A^{2009}

This describes the approximate long term behaviour of almost everything ...

The Theory side.

3. Do all of this in a coordinate-free way?

4. Do all this over "other sets of numbers".

5. Hidden agenda: Learn the basic pure-math processes of abstraction, generalization, definitions theorems, proofs, notation, logic.

I should have added to the hidden agenda:
 * Tell right from wrong,
 * Tell grammatical non-grammatical.
 * Tell well-written from badly-written.

(maybe by the end of college:
 * Tell understood from not-understood)

2. Go over the "About" handout.

3 The real numbers: a set $\mathbb{R}$ with two binary ops $+$ & $\times$ and two special elements 0 & 1 s.t.

R1 $a + b = b + a$ $ab = ba$

R2   ASSOC.   R4 negatives & inverses

R3   0, 1   R5 Distributivity.

Much of algebra, though not all, follows:

Follows: $(a+b)(a-b) = a^2 - b^2$

Doesn't follow: $\forall a \; \exists x$ s.t. $a = x^2$ or $a = -x^2$

<u>Def</u> A Field F

<u>Examples</u> 1. The reals $\mathbb{R}$.

2. The rationals $\mathbb{Q}$

3. The complex numbers $\mathbb{C} = \{a + b\sqrt{-1}\}$

4. $0, 1$ with

+	0	1
0	0	1
1	1	0

·	0	1
0	0	0
1	0	1

5. $0, 1, 2, 3, 4, 5, 6$ with a funny def. of $+, \times$.

Proofs...

4. <u>Thm</u> 1. $a + b = c + b \Rightarrow a = c$

~~~~ don't line

2. $a \cdot b = c \cdot b$, $b \neq 0 \Rightarrow a = c$

3. If $0'$ is like 0, then $0' = 0$

4. If $1'$ is like 1, then $1' = 1$

5. If $a + b = 0 = a + b'$ then $b = b'$
   (so we can define $-a$)

6. If $a \neq 1$ & $ab = 1 = ab' \Rightarrow b = b'$
   (so we can define a^{-1}

7. $-(-a) = a$, $(a^{-1})^{-1} = a$

def: subtraction $a - b$, division a/b when $b \neq 0$

8. $a \cdot 0 = 0$

9. There's no 0^{-1}

That's hard

10. $(-a) \cdot b = a \cdot (-b) = -(a \cdot b)$

11. $(-a)(-b) = a \cdot b$

4. <u>Def</u> Characteristic