

and $K \cap H = \{e\}$

Claim. IF $K \triangleleft KH$, $H \triangleleft KH$, Then $KH = K \times H$.

$$K \rightarrow KH \rightarrow KH/H \cong K$$

$$k_1 h_1 = k_2 h_2 \Rightarrow k_2^{-1} k_1 = h_1 h_2^{-1} \Rightarrow k_1 = k_2, h_1 = h_2$$

$$h k = k h^k = k^{h^{-1}} h \Rightarrow h^k = h \Rightarrow [h, k] = e.$$

$$h^k h^{-1} = k^{-1} k^{h^{-1}} \Leftrightarrow k^{-1} h k h^{-1} \in H \cap K = \{e\}$$