

Pensieve Header: Computing the maximal order of a permutation in SS_n .

```
$RecursionLimit = 10 000;
MaxOrder[n_] := MaxOrder[n, 2];
MaxOrder[n_, p_] /; p > n := 1;
MaxOrder[n_, p_] := MaxOrder[n, p] = (
  ++it;
  Max[Table[
    p^k * MaxOrder[n - (p^k /. 1 → 0), NextPrime[p]],
    {k, 0, Floor[Log[p, n]]}
  ]]
)
it = 0; {MaxOrder[10 000], it}

{83 724 831 478 115 383 622 262 752 790 863 851 979 860 821 403 995 464 606 889 515 707 038 262 291 \
  218 453 286 312 511 300 740 953 193 197 240 510 692 638 318 426 636 506 681 182 400, 6 501 964}

FactorInteger[
  83 724 831 478 115 383 622 262 752 790 863 851 979 860 821 403 995 464 606 889 515 707 038 262 291 \
  218 453 286 312 511 300 740 953 193 197 240 510 692 638 318 426 636 506 681 182 400]

{{2, 6}, {3, 4}, {5, 2}, {7, 2}, {11, 2}, {13, 2}, {17, 1}, {19, 1}, {23, 1},
 {29, 1}, {31, 1}, {37, 1}, {41, 1}, {43, 1}, {47, 1}, {53, 1}, {59, 1}, {61, 1},
 {67, 1}, {71, 1}, {73, 1}, {79, 1}, {83, 1}, {89, 1}, {97, 1}, {101, 1},
 {103, 1}, {107, 1}, {109, 1}, {113, 1}, {127, 1}, {131, 1}, {137, 1}, {139, 1},
 {149, 1}, {151, 1}, {157, 1}, {163, 1}, {167, 1}, {173, 1}, {179, 1}, {181, 1},
 {191, 1}, {193, 1}, {197, 1}, {199, 1}, {211, 1}, {223, 1}, {227, 1}, {229, 1},
 {233, 1}, {239, 1}, {241, 1}, {251, 1}, {257, 1}, {263, 1}, {269, 1}, {271, 1},
 {277, 1}, {281, 1}, {283, 1}, {293, 1}, {307, 1}, {311, 1}, {313, 1}, {317, 1}}

Table[Prime[k], {k, 70}]

{2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73,
  79, 83, 89, 97, 101, 103, 107, 109, 113, 127, 131, 137, 139, 149, 151, 157, 163,
  167, 173, 179, 181, 191, 193, 197, 199, 211, 223, 227, 229, 233, 239, 241, 251,
  257, 263, 269, 271, 277, 281, 283, 293, 307, 311, 313, 317, 331, 337, 347, 349}

Print[# → {m = MaxOrder[#], FactorInteger[m]}] & /@ Range[100];

1 → {1, {{1, 1}}}
2 → {2, {{2, 1}}}
3 → {3, {{3, 1}}}
4 → {4, {{2, 2}}}
5 → {6, {{2, 1}, {3, 1}}}
6 → {6, {{2, 1}, {3, 1}}}
7 → {12, {{2, 2}, {3, 1}}}
8 → {15, {{3, 1}, {5, 1}}}
9 → {20, {{2, 2}, {5, 1}}}
10 → {30, {{2, 1}, {3, 1}, {5, 1}}}
11 → {30, {{2, 1}, {3, 1}, {5, 1}}}
12 → {60, {{2, 2}, {3, 1}, {5, 1}}}
```

13 → {60, {{2, 2}, {3, 1}, {5, 1}}}
 14 → {84, {{2, 2}, {3, 1}, {7, 1}}}
 15 → {105, {{3, 1}, {5, 1}, {7, 1}}}
 16 → {140, {{2, 2}, {5, 1}, {7, 1}}}
 17 → {210, {{2, 1}, {3, 1}, {5, 1}, {7, 1}}}
 18 → {210, {{2, 1}, {3, 1}, {5, 1}, {7, 1}}}
 19 → {420, {{2, 2}, {3, 1}, {5, 1}, {7, 1}}}
 20 → {420, {{2, 2}, {3, 1}, {5, 1}, {7, 1}}}
 21 → {420, {{2, 2}, {3, 1}, {5, 1}, {7, 1}}}
 22 → {420, {{2, 2}, {3, 1}, {5, 1}, {7, 1}}}
 23 → {840, {{2, 3}, {3, 1}, {5, 1}, {7, 1}}}
 24 → {840, {{2, 3}, {3, 1}, {5, 1}, {7, 1}}}
 25 → {1260, {{2, 2}, {3, 2}, {5, 1}, {7, 1}}}
 26 → {1260, {{2, 2}, {3, 2}, {5, 1}, {7, 1}}}
 27 → {1540, {{2, 2}, {5, 1}, {7, 1}, {11, 1}}}
 28 → {2310, {{2, 1}, {3, 1}, {5, 1}, {7, 1}, {11, 1}}}
 29 → {2520, {{2, 3}, {3, 2}, {5, 1}, {7, 1}}}
 30 → {4620, {{2, 2}, {3, 1}, {5, 1}, {7, 1}, {11, 1}}}
 31 → {4620, {{2, 2}, {3, 1}, {5, 1}, {7, 1}, {11, 1}}}
 32 → {5460, {{2, 2}, {3, 1}, {5, 1}, {7, 1}, {13, 1}}}
 33 → {5460, {{2, 2}, {3, 1}, {5, 1}, {7, 1}, {13, 1}}}
 34 → {9240, {{2, 3}, {3, 1}, {5, 1}, {7, 1}, {11, 1}}}
 35 → {9240, {{2, 3}, {3, 1}, {5, 1}, {7, 1}, {11, 1}}}
 36 → {13 860, {{2, 2}, {3, 2}, {5, 1}, {7, 1}, {11, 1}}}
 37 → {13 860, {{2, 2}, {3, 2}, {5, 1}, {7, 1}, {11, 1}}}
 38 → {16 380, {{2, 2}, {3, 2}, {5, 1}, {7, 1}, {13, 1}}}
 39 → {16 380, {{2, 2}, {3, 2}, {5, 1}, {7, 1}, {13, 1}}}
 40 → {27 720, {{2, 3}, {3, 2}, {5, 1}, {7, 1}, {11, 1}}}
 41 → {30 030, {{2, 1}, {3, 1}, {5, 1}, {7, 1}, {11, 1}, {13, 1}}}
 42 → {32 760, {{2, 3}, {3, 2}, {5, 1}, {7, 1}, {13, 1}}}
 43 → {60 060, {{2, 2}, {3, 1}, {5, 1}, {7, 1}, {11, 1}, {13, 1}}}
 44 → {60 060, {{2, 2}, {3, 1}, {5, 1}, {7, 1}, {11, 1}, {13, 1}}}
 45 → {60 060, {{2, 2}, {3, 1}, {5, 1}, {7, 1}, {11, 1}, {13, 1}}}
 46 → {60 060, {{2, 2}, {3, 1}, {5, 1}, {7, 1}, {11, 1}, {13, 1}}}
 47 → {120 120, {{2, 3}, {3, 1}, {5, 1}, {7, 1}, {11, 1}, {13, 1}}

48 → {120 120, {{2, 3}, {3, 1}, {5, 1}, {7, 1}, {11, 1}, {13, 1}}}
 49 → {180 180, {{2, 2}, {3, 2}, {5, 1}, {7, 1}, {11, 1}, {13, 1}}}
 50 → {180 180, {{2, 2}, {3, 2}, {5, 1}, {7, 1}, {11, 1}, {13, 1}}}
 51 → {180 180, {{2, 2}, {3, 2}, {5, 1}, {7, 1}, {11, 1}, {13, 1}}}
 52 → {180 180, {{2, 2}, {3, 2}, {5, 1}, {7, 1}, {11, 1}, {13, 1}}}
 53 → {360 360, {{2, 3}, {3, 2}, {5, 1}, {7, 1}, {11, 1}, {13, 1}}}
 54 → {360 360, {{2, 3}, {3, 2}, {5, 1}, {7, 1}, {11, 1}, {13, 1}}}
 55 → {360 360, {{2, 3}, {3, 2}, {5, 1}, {7, 1}, {11, 1}, {13, 1}}}
 56 → {360 360, {{2, 3}, {3, 2}, {5, 1}, {7, 1}, {11, 1}, {13, 1}}}
 57 → {471 240, {{2, 3}, {3, 2}, {5, 1}, {7, 1}, {11, 1}, {17, 1}}}
 58 → {510 510, {{2, 1}, {3, 1}, {5, 1}, {7, 1}, {11, 1}, {13, 1}, {17, 1}}}
 59 → {556 920, {{2, 3}, {3, 2}, {5, 1}, {7, 1}, {13, 1}, {17, 1}}}
 60 → {1 021 020, {{2, 2}, {3, 1}, {5, 1}, {7, 1}, {11, 1}, {13, 1}, {17, 1}}}
 61 → {1 021 020, {{2, 2}, {3, 1}, {5, 1}, {7, 1}, {11, 1}, {13, 1}, {17, 1}}}
 62 → {1 141 140, {{2, 2}, {3, 1}, {5, 1}, {7, 1}, {11, 1}, {13, 1}, {19, 1}}}
 63 → {1 141 140, {{2, 2}, {3, 1}, {5, 1}, {7, 1}, {11, 1}, {13, 1}, {19, 1}}}
 64 → {2 042 040, {{2, 3}, {3, 1}, {5, 1}, {7, 1}, {11, 1}, {13, 1}, {17, 1}}}
 65 → {2 042 040, {{2, 3}, {3, 1}, {5, 1}, {7, 1}, {11, 1}, {13, 1}, {17, 1}}}
 66 → {3 063 060, {{2, 2}, {3, 2}, {5, 1}, {7, 1}, {11, 1}, {13, 1}, {17, 1}}}
 67 → {3 063 060, {{2, 2}, {3, 2}, {5, 1}, {7, 1}, {11, 1}, {13, 1}, {17, 1}}}
 68 → {3 423 420, {{2, 2}, {3, 2}, {5, 1}, {7, 1}, {11, 1}, {13, 1}, {19, 1}}}
 69 → {3 423 420, {{2, 2}, {3, 2}, {5, 1}, {7, 1}, {11, 1}, {13, 1}, {19, 1}}}
 70 → {6 126 120, {{2, 3}, {3, 2}, {5, 1}, {7, 1}, {11, 1}, {13, 1}, {17, 1}}}
 71 → {6 126 120, {{2, 3}, {3, 2}, {5, 1}, {7, 1}, {11, 1}, {13, 1}, {17, 1}}}
 72 → {6 846 840, {{2, 3}, {3, 2}, {5, 1}, {7, 1}, {11, 1}, {13, 1}, {19, 1}}}
 73 → {6 846 840, {{2, 3}, {3, 2}, {5, 1}, {7, 1}, {11, 1}, {13, 1}, {19, 1}}}
 74 → {6 846 840, {{2, 3}, {3, 2}, {5, 1}, {7, 1}, {11, 1}, {13, 1}, {19, 1}}}
 75 → {6 846 840, {{2, 3}, {3, 2}, {5, 1}, {7, 1}, {11, 1}, {13, 1}, {19, 1}}}
 76 → {8 953 560, {{2, 3}, {3, 2}, {5, 1}, {7, 1}, {11, 1}, {17, 1}, {19, 1}}}
 77 → {9 699 690, {{2, 1}, {3, 1}, {5, 1}, {7, 1}, {11, 1}, {13, 1}, {17, 1}, {19, 1}}}
 78 → {12 252 240, {{2, 4}, {3, 2}, {5, 1}, {7, 1}, {11, 1}, {13, 1}, {17, 1}}}
 79 → {19 399 380, {{2, 2}, {3, 1}, {5, 1}, {7, 1}, {11, 1}, {13, 1}, {17, 1}, {19, 1}}}
 80 → {19 399 380, {{2, 2}, {3, 1}, {5, 1}, {7, 1}, {11, 1}, {13, 1}, {17, 1}, {19, 1}}}
 81 → {19 399 380, {{2, 2}, {3, 1}, {5, 1}, {7, 1}, {11, 1}, {13, 1}, {17, 1}, {19, 1}}}
 82 → {19 399 380, {{2, 2}, {3, 1}, {5, 1}, {7, 1}, {11, 1}, {13, 1}, {17, 1}, {19, 1}}

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83 → {38 798 760, {{2, 3}, {3, 1}, {5, 1}, {7, 1}, {11, 1}, {13, 1}, {17, 1}, {19, 1}}}
84 → {38 798 760, {{2, 3}, {3, 1}, {5, 1}, {7, 1}, {11, 1}, {13, 1}, {17, 1}, {19, 1}}}
85 → {58 198 140, {{2, 2}, {3, 2}, {5, 1}, {7, 1}, {11, 1}, {13, 1}, {17, 1}, {19, 1}}}
86 → {58 198 140, {{2, 2}, {3, 2}, {5, 1}, {7, 1}, {11, 1}, {13, 1}, {17, 1}, {19, 1}}}
87 → {58 198 140, {{2, 2}, {3, 2}, {5, 1}, {7, 1}, {11, 1}, {13, 1}, {17, 1}, {19, 1}}}
88 → {58 198 140, {{2, 2}, {3, 2}, {5, 1}, {7, 1}, {11, 1}, {13, 1}, {17, 1}, {19, 1}}}
89 → {116 396 280, {{2, 3}, {3, 2}, {5, 1}, {7, 1}, {11, 1}, {13, 1}, {17, 1}, {19, 1}}}
90 → {116 396 280, {{2, 3}, {3, 2}, {5, 1}, {7, 1}, {11, 1}, {13, 1}, {17, 1}, {19, 1}}}
91 → {116 396 280, {{2, 3}, {3, 2}, {5, 1}, {7, 1}, {11, 1}, {13, 1}, {17, 1}, {19, 1}}}
92 → {116 396 280, {{2, 3}, {3, 2}, {5, 1}, {7, 1}, {11, 1}, {13, 1}, {17, 1}, {19, 1}}}
93 → {140 900 760, {{2, 3}, {3, 2}, {5, 1}, {7, 1}, {11, 1}, {13, 1}, {17, 1}, {23, 1}}}
94 → {140 900 760, {{2, 3}, {3, 2}, {5, 1}, {7, 1}, {11, 1}, {13, 1}, {17, 1}, {23, 1}}}
95 → {157 477 320, {{2, 3}, {3, 2}, {5, 1}, {7, 1}, {11, 1}, {13, 1}, {19, 1}, {23, 1}}}
96 → {157 477 320, {{2, 3}, {3, 2}, {5, 1}, {7, 1}, {11, 1}, {13, 1}, {19, 1}, {23, 1}}}
97 → {232 792 560, {{2, 4}, {3, 2}, {5, 1}, {7, 1}, {11, 1}, {13, 1}, {17, 1}, {19, 1}}}
98 → {232 792 560, {{2, 4}, {3, 2}, {5, 1}, {7, 1}, {11, 1}, {13, 1}, {17, 1}, {19, 1}}}
99 → {232 792 560, {{2, 4}, {3, 2}, {5, 1}, {7, 1}, {11, 1}, {13, 1}, {17, 1}, {19, 1}}}
100 → {232 792 560, {{2, 4}, {3, 2}, {5, 1}, {7, 1}, {11, 1}, {13, 1}, {17, 1}, {19, 1}}}

```

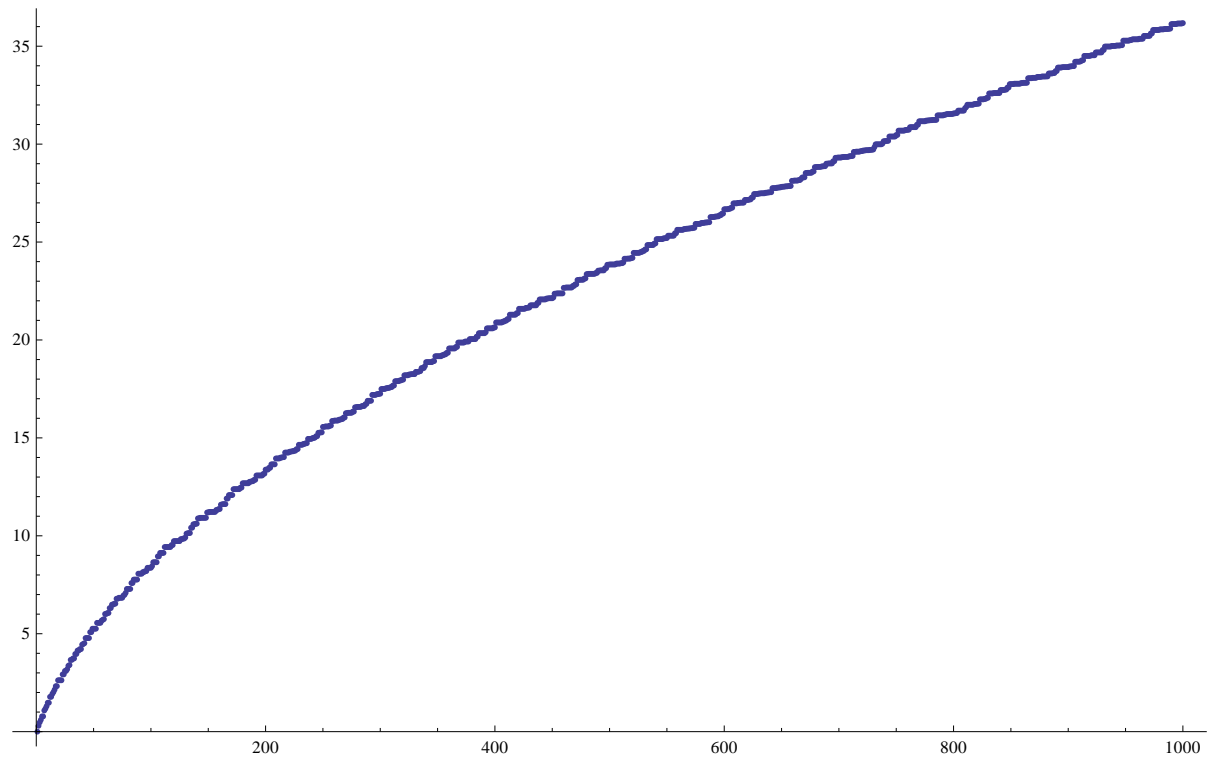
Timing[{m = MaxOrder[1000], FactorInteger[m]}]

```

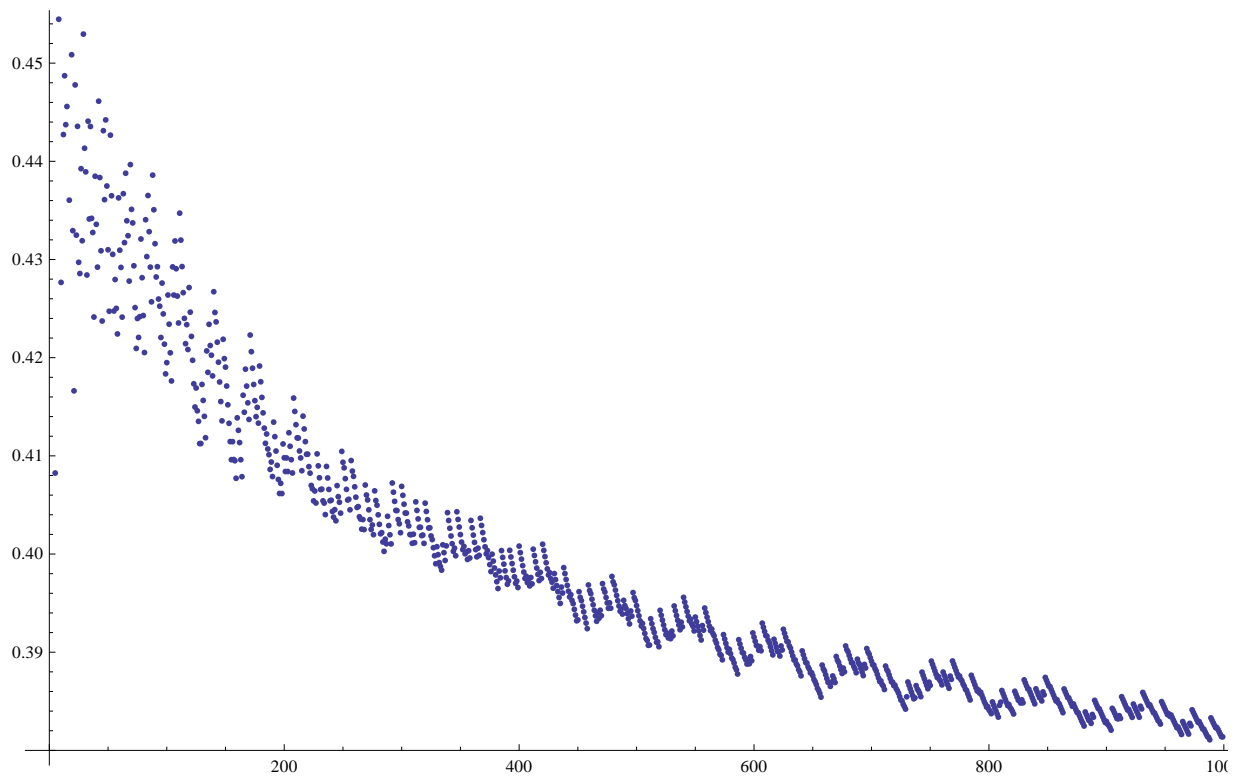
{5.647, {1 516 385 558 956 728 808 659 224 171 023 365 600,
  {{2, 5}, {3, 3}, {5, 2}, {7, 2}, {11, 1}, {13, 1}, {17, 1}, {19, 1},
  {23, 1}, {29, 1}, {31, 1}, {37, 1}, {41, 1}, {43, 1}, {47, 1}, {53, 1},
  {59, 1}, {61, 1}, {67, 1}, {71, 1}, {73, 1}, {83, 1}, {89, 1}}}}

```

```
ListPlot[Log[10, MaxOrder[#]] & /@ Range[1000]]
```



```
ListPlot[(Log[MaxOrder[#]] / (Sqrt[#] Log[#])) & /@ Range[2, 1000]]
```



```
Log[2] / 2 // N
```

```
0.346574
```

```
n = 10 000; Log[MaxOrder[n]] / (Sqrt[n] Log[n]) // N
```

```
0.342307
```

```
Table[MaxOrder[n], {n, 20}]
```

```
{1, 2, 3, 4, 6, 6, 12, 15, 20, 30, 30, 60, 60, 84, 105, 140, 210, 210, 420, 420}
```