Dror Bar-Natan: Classes: 2011-12: Math 1100 Algebra I:

Running the JCF Programs

```
SetDirectory["C:\\drorbn\\AcademicPensieve\\Classes\\11-1100"];
<< JCF-Program.m
```

Matrix 1 - 3x3, 3 eigenvalues.

n = 3; AA = $\begin{pmatrix} 3 & 0 & 0 \\ 4 & -2 & -6 \\ -2 & 0 & 1 \end{pmatrix}$;

```
PP = QQ = II = IdentityMatrix[n];
MM = x II - AA;  NN = PP.MM.QQ;
```

```
SwapRows[1, 2]
```

```
SwapColumns[2, 3]
```

```
? PolynomialExtendedGCD
```

PolynomialExtendedGCD[$poly_1$, $poly_2$, x] gives the extended GCD of $poly_1$ and $poly_2$ treated as univariate polynomials in x.
PolynomialExtendedGCD[$poly_1$, $poly_2$, x, Modulus –> p] gives the extended GCD over the integers mod prime p. >>

```
GCDTrick[{1, 2}, 1]
```

```
GCDTrick[1, {1, 2}]
```

```
GCDTrick[1, {1, 3}]
```

```
GCDTrick[{1, 3}, 1]
```

```
GCDTrick[2, {2, 3}]
```

```
GCDTrick[{2, 3}, 2]
```

```
GCDTrick[2, {2, 3}]
```

SplitToSum[1, 3, $(-3 + x)$, $(-2 + x + x^2)$]

Factor[$-2 + x + x^2$]

$(-1 + x)\ (2 + x)$

SplitToSum[2, 3, $(-1 + x)$, $(2 + x)$]

```
MatrixForm /@ {Coefficient[NN, x, 1], BB = -Coefficient[NN, x, 0]}
```

$$\left\{\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix},\begin{pmatrix}3&0&0\\0&1&0\\0&0&-2\end{pmatrix}\right\}$$

```
(CC = Sum[
    MatrixPower[BB, k].Coefficient[PP, x, k],
    {k, 0, n}
  ]) // MatrixForm
```

$$\begin{pmatrix}-\frac{10}{3}&0&0\\-2&0&-2\\0&-\frac{5}{2}&-5\end{pmatrix}$$

```
CC.AA.Inverse[CC] // MatrixForm
```

$$\begin{pmatrix}3&0&0\\0&1&0\\0&0&-2\end{pmatrix}$$

Matrix 2 - 3x3, one Jordan block.

$$\texttt{n = 3; AA = }\begin{pmatrix}-\frac{5}{2}&-11&\frac{9}{2}\\-\frac{1}{2}&1&\frac{1}{2}\\-\frac{19}{2}&-16&\frac{21}{2}\end{pmatrix};$$

```
PP = QQ = II = IdentityMatrix[n];
MM = x II - AA; NN = PP.MM.QQ;
```

```
GCDTrick[1, {1, 2}]
```

```
GCDTrick[1, {1, 3}]
```

```
GCDTrick[{1, 2}, 1]
```

```
GCDTrick[{1, 3}, 1]
```

```
GCDTrick[2, {2, 3}]
```

```
GCDTrick[{2, 3}, 2]
```

```
JordanTrick[2, 3, x - 3, 3]
```

```
JordanTrick[1, 2, x - 3, 2]
```

```
MatrixForm /@ {Coefficient[NN, x, 1], BB = -Coefficient[NN, x, 0]}
```

$$\left\{\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix},\begin{pmatrix}3&0&0\\-1&3&0\\0&-1&3\end{pmatrix}\right\}$$

```
(CC = Sum[
    MatrixPower[BB, k].Coefficient[PP, x, k],
    {k, 0, n}
   ]) // MatrixForm
```

$$\begin{pmatrix} -2 & 3 & 1 \\ -1 & 7 & 0 \\ -1 & 9 & 0 \end{pmatrix}$$

```
CC.AA.Inverse[CC] // MatrixForm
```

$$\begin{pmatrix} 3 & 0 & 0 \\ -1 & 3 & 0 \\ 0 & -1 & 3 \end{pmatrix}$$

Matrix 3 - 4x4, mixed Jordan form.

$$n = 4;\ AA = \begin{pmatrix} 1 & -2 & 0 & -2 \\ \frac{1}{4} & \frac{5}{2} & 0 & \frac{3}{2} \\ \frac{5}{2} & 5 & 2 & 3 \\ \frac{1}{4} & \frac{1}{2} & 0 & \frac{3}{2} \end{pmatrix};$$

```
PP = QQ = II = IdentityMatrix[n];
MM = x II - AA; NN = PP.MM.QQ;

GCDTrick[{1, 2}, 1]

GCDTrick[1, {1, 2}]

GCDTrick[1, {1, 4}]

GCDTrick[{1, 3}, 1]

GCDTrick[{1, 4}, 1]

GCDTrick[2, {2, 3}]

GCDTrick[2, {2, 4}]

GCDTrick[{2, 3}, 2]

GCDTrick[{2, 4}, 2]

GCDTrick[3, {3, 4}]

SwapBoth[1, 3]

SplitToSum[2, 4, (-1 + x), (-2 + x)^2]

SwapBoth[1, 2]

JordanTrick[3, 4, x - 2, 2]
```

```
MatrixForm /@ {Coefficient[NN, x, 1], BB = -Coefficient[NN, x, 0]}
```

$$\left\{\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{pmatrix}, \begin{pmatrix}2&0&0&0\\0&1&0&0\\0&0&2&0\\0&0&-1&2\end{pmatrix}\right\}$$

```
(CC = Sum[
    MatrixPower[BB, k].Coefficient[PP, x, k],
    {k, 0, n}
   ]) // MatrixForm
```

$$\begin{pmatrix}3&2&1&0\\-\frac{1}{2}&-1&0&1\\-\frac{1}{2}&-1&0&-1\\-\frac{1}{2}&0&0&0\end{pmatrix}$$

```
CC.AA.Inverse[CC] // MatrixForm
```

$$\begin{pmatrix}2&0&0&0\\0&1&0&0\\0&0&2&0\\0&0&-1&2\end{pmatrix}$$