September 16, hours 2-3: Basics of Topological Spaces

September-15-10
7:54 PM

APUS wants you!

Remember blackboard shots!

Go over the "about" handout.

Read along: Munkres sections 12-17. Forgotten.

Theorem $f: \mathbb{R}^n \to \mathbb{R}^m$ is continuous iff for every open set $U \subset \mathbb{R}^m$, $f^{-1}(U)$ is also open. $\cap \mathbb{R}^n$

Properties of open sets:

1. $\emptyset, \mathbb{R}$ 2. $\bigcup$ 3 Finite $\bigcap$.

on board

Definition 1. A topological space

2. Continuous function $f: X \to Y$.

Theorem The composition of continuous functions is continuous.

Examples The discrete and trivial topologies, continuous functions $f: \mathbb{R}_{discrete, trivial} \to \mathbb{R}_{standard}$

$f: \mathbb{R}_{std} \to X_{discrete, trivial}$.

Definition $\mathcal{T}_1 \supset \mathcal{T}_2$ is "$\mathcal{T}_1$ is finer / bigger / stronger than $\mathcal{T}_2$" while "$\mathcal{T}_2$ is coarser / smaller / weaker" than $\mathcal{T}_1$.

skipped

The identity is continuous iff it goes from the finer topology to the weaker one.

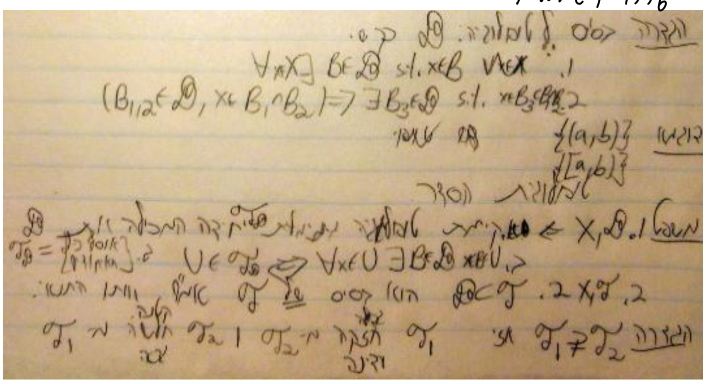
Lesson: Must say something about homeomorphisms, around here!

March 1996

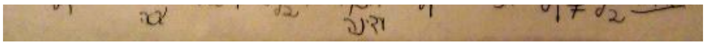
Basis for a topology $\{(a,b)\}$, $\{[a,b)\}$

Thm Given a basis for a topology, a. there exists a unique minimal topology containing it. b. $U \in \mathcal{T}_{\mathcal{B}} \iff \forall x \in U\ \exists B \in \mathcal{B} \ldots$ c. $\mathcal{T}_{\mathcal{B}}$ is the collection of all unions of

stated but not proven



ת | of all unions of elements of $\mathcal{D}$



should have said: Note that this is the same word as in linear algebra, but a different notion.

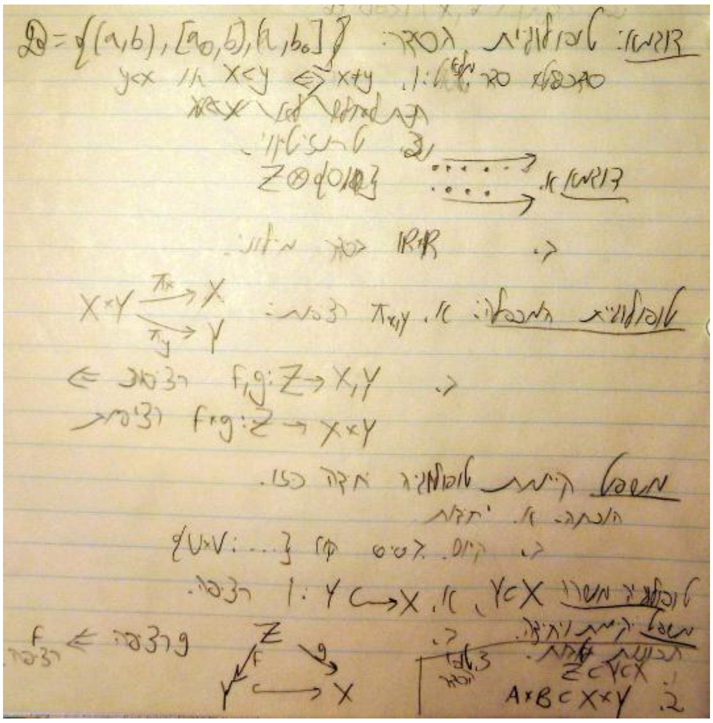
done line

<u>Claim</u> $\mathbb{R}_\ell = (\mathbb{R}, \mathcal{T}_{\{[a,b)\}})$ "the lower limit topology" is finer than $\mathbb{R}_{std}$.

Examples/constructions of further topologies.

1. Order
2. Product
3. Subspace.
4. Compatibilities:
   - Sub & Sub
   - Sub & product
   - Sub & order, in the convex case.

<u>Example</u> The dictionary-order topology on I^2 is different than the topology induced on I^2 from the dictionary order topology of $\mathbb{R}^2$



<u>דוגמה</u>: טופולוגית הסדר: $\mathcal{D} = \{(a,b), [a_0,b), (a,b_0]\}$

$x\neq y \Leftrightarrow x<y$ או $y<x$

<u>דוגמה</u> א. $\mathbb{Z}\otimes\{0,1\}$

ב. $\mathbb{R}\times\mathbb{R}$ סדר מילוני.

<u>טופולוגית המכפלה</u>: א. $\pi_{x,y}$ רציפות: $X\times Y \xrightarrow{\pi_x} X$, $X\times Y \xrightarrow{\pi_y} Y$

ב. $f,g: Z\to X,Y$ רציפות $\Leftarrow$ $f\times g: Z\to X\times Y$ רציפה

<u>משפט</u> קיימת טופולוגיה אחת כזו.

הוכחה. א. יחידות

ב. קיום. בסיס $\{U\times V: \dots\}$

<u>טופולוגית תת־מרחב</u> $Y\subset X$, א. $i: Y\hookrightarrow X$ רציפה.

ב. $f: Z\to Y$, $g: Z\to X$: g רציפה $\Leftarrow$ f רציפה

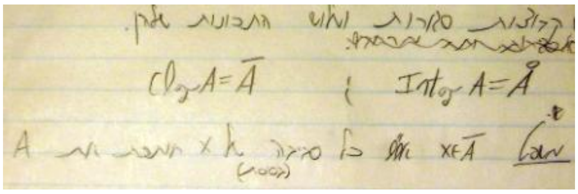
<u>משפט</u> קיימת ויחידה.

תכונות אחרות. 1. $Z\subset Y\subset X$ 2. $A\times B\subset X\times Y$

Closed set & their basics

closure & interior.

Condition for $x\in\bar{A}$.



קבוצות סגורות ואלו התכונות שלהן.

$Cl_{\mathcal{T}} A = \bar{A}$; $Int_{\mathcal{T}} A = \mathring{A}$

<u>למה</u> $x\in\bar{A}$ אמ"מ כל סביבה (פתוחה) של x חותכת את A