October 21, hours 16-17: Connectedness, path-connectedness

October-18-10
6:25 PM

HW. HW3 will be returned Monday or tomorrow/ 11-12 @ math lounge.

The TT:

Read Along. Munkres 23, 24, 26, 27.

Riddle Along. Can you colour the points of $\mathbb{R}^2$ in 4 colours s.t. no pair of distance exactly one are of the same colour?

Theorem. If A is connected & $A \subset B \subset \bar{A}$, B is too.

Pf Assume C is clopen in B, $C \cap A \neq \emptyset$. Then $C \supset A$ so $cl_X C \supset \bar{A} \supset B$, so $cl_X C \cap B = B$, so $cl_B C = B$, so $C = B$.

Theorem. If $\forall \alpha$ X_α is connected, then $\prod X_\alpha$ is connected.

Example. $\mathbb{R}^\omega = \left\{ \begin{matrix} \text{bndd} \\ \text{seqs} \end{matrix} \right\} \cup \left\{ \begin{matrix} \text{unbndd} \\ \text{seqs.} \end{matrix} \right\}$ is a box-separation.

Def. Path-connected; Path-connected $\Rightarrow$ connected
(1. Proof from defs.
2. Lemma: If X is connected, so is $f(X)$)

The topologist's sine curve

A product of path connected spaces is path-connected.

done line

Def. Cover, open cover, Compact.

Thm. A continuous function to $\mathbb{R}$ on a compact set is bounded.

Thm. A closed subset of a compact space is compact.

Thm. A compact subset of a T_2 space is closed.

Thm. $[0,1]$ is compact.

Thm. $[0,1]$ is compact.

Corollary. A subset of $\mathbb{R}$ is compact iff it is closed and bounded.