# October 18, hour 15: Connectedness

October-17-10
4:18 PM

HW3 due now, HW4 on web by midnight, All about $\prod$ on Thursday.

**Read Along.** Munkres 23, 24.

**Riddle Along.** Show that on any Potato you can find two congruent curves.

**Connectedness.** Separation, connectedness, clopen sets.

**The I.V.T.** If $X$ is connected, $f: X \to \mathbb{R}$ cont., $f(x_0)<0$, $f(x_1)>0 \Rightarrow \exists x$ s.t. $f(x)=0$.

**Theorem** $I=[0,1]$ is connected.

**Proof.** Assume $0 \in A \subset I$ is clopen. Let

$$G=\{x : [0,x] \subset A\} \qquad g=\sup G$$

1. $g>0$ 2. $g \nless 1$ 3. $1 \in G$.

**Theorem.** If $A_\alpha \subset X$ are connected, $\bigcap A_\alpha \neq \emptyset$, then $\bigcup A_\alpha$ is connected.

**Theorem.** $A \subset \mathbb{R}$ is connected iff it is an interval, or a ray, or the whole thing. [I.e., if it is "convex"]

done line.

**Theorem.** If $A$ is connected & $A \subset B \subset \bar{A}$, $B$ is too.

**Pf** Assume $C$ is clopen in $B$, $C \cap A \neq \emptyset$. Then $C \supset A$ so $cl_X C \supset \bar{A} \supset B$, so $cl_X C \cap B = B$, so $cl_B C = B$, so $C = B$.

**Theorem.** If $\forall \alpha$ $X_\alpha$ is connected, then $\prod X_\alpha$ is connected.

**Example.** $\mathbb{R}^\omega = \left\{\begin{matrix}\text{bndd}\\ \text{seqs}\end{matrix}\right\} \cup \left\{\begin{matrix}\text{unbndd}\\ \text{seqs.}\end{matrix}\right\}$ is a box-separation.