October 21, hour 18: Braids, Orders pq and 12, Solvable groups

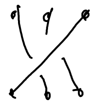
October-19-10
8:52 AM

Read Along. Selick 1.8, 1.10, 1.11.

Chandler Davis' Lecture.

Riddle Along. $\boxed{\forall \alpha \in \mathbb{R}\ \exists a_i \in \mathbb{Q} \text{ s.t. } a_i \to \alpha}$ $\boxed{\mathbb{Q} \cap [0,\alpha]}$ what do these solve?

Aside. Go over MaxPermOrder.

Big Example. $B_n = \pi_1\left(\left(\mathbb{C}^2 - \{\text{diags}\}\right)/S_n\right) =$ 

done but review needed.

$B_n = \left\langle \sigma_1, \dots \sigma_{n-1} : \begin{array}{l} \sigma_i\sigma_{i+1}\sigma_i = \sigma_{i+1}\sigma_i\sigma_{i+1} \\ \sigma_i\sigma_j = \sigma_j\sigma_i \quad |i-j|>1 \end{array} \right\rangle$ } an aside on free groups, generators & relations.

$\pi : B_n \longrightarrow S_n \qquad PB_n = \ker \pi$

Two reasons why I like this one:
1. knotted $20's.
2. Borromean.

$PB_n \triangleleft B_n$ yet not $B_n = S_n \ltimes PB_n$

$\rho : PB_n \to PB_{n-1} \qquad \ker \rho = F_{n-1}$ and

$PB_n = PB_{n-1} \ltimes F_{n-1} = ((\dots(F_1 \ltimes F_2) \ltimes F_3 \dots \ltimes F_{n-2}) \ltimes F_{n-1})$

not done

Groups of order 21. $\mathbb{Z}/21$, $\mathbb{Z}/7 \rtimes \mathbb{Z}/3 = \langle x \rangle \rtimes \langle y \rangle$

$Aut(\mathbb{Z}/7) = \mathbb{Z}/6 = \langle \phi_3 \rangle$; $\phi_3(x) = x^3$; $x^y = x$ or x^2 or x^4 (isomorphic)

(iso: if $x^y = x^2$ & $\bar{y} = y^2$ then $x^{\bar{y}} = x^4$)

Groups of order 12. If $|G| = 12$, $P_4 = \mathbb{Z}/4$ or $(\mathbb{Z}/2)^2$, $P_3 = \mathbb{Z}/3$, and at least one of these is normal, for there's not enough room for 4 P_3 & 3 P_4's. So G is a semi-direct product.

$\mathbb{Z}/12$, $\mathbb{Z}/2 \times \mathbb{Z}/6$, A_4, D_{12}, $\mathbb{Z}/3 \rtimes \mathbb{Z}/4$

The most fun case is $\mathbb{Z}/3 \curvearrowright (\mathbb{Z}/2)^2$, giving A_4.

Solvable Groups. <u>Def</u> G is solvable if all quotients in its Jordan-Hölder series are Abelian.

<u>Thm</u> 1. If $N \triangleleft G$, G is solvable iff N & G/N are.

2. If $H < G$ and G is solvable, so is H.

$A \triangleleft B \quad H \cap A \triangleleft H \cap B$? ✓ $\quad \dfrac{H \cap B}{H \cap A} \longrightarrow B/A$ by $[b]_{H\cap A} \to [b]_A$ is injective.