Groups of Order pq

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D&F pp 143 & pp 181

subgroups of G of order q^α, where α is the maximal so that $q^\alpha \mid |G|$.

$|G| = pq$, p & q primes, $p < q$.

Let $P \in Syl_p(G)$ & $Q \in Syl_q(G)$. By Sylow 2 Q is normal — the size of its conjugation orbit divides pq and is 1 mod q, so it must be 1.

If P is also normal, it must commute with every $y \in Q$. Otherwise conjugation by y would be an automorphism of P whose order divides $q = |Q|$ and $p-1 = |Aut(P)|$, and this is impossible. So P and Q commute and G is overall commutative.

If P isn't normal, $n_p(G) > 1$ yet it is 1 mod p and it divides $|G|$ and hence it divides q. So $n_p(G) = q$ and $p \mid q-1$.

<u>Examples</u> $15 = 3 \cdot 5$ $\quad 3 \nmid 4 = 5-1$

$6 = 2 \cdot 3 \quad 2 \mid 2 = 3-1 \quad 21 = 3 \cdot 7 \quad 3 \mid 6 = 7-1$