December 7, hours 35-36: Unboxing the Jordan Canonical Form

December-04-10
8:07 AM

Class outline: 0. Admin 1. Cayley-Hamilton

2. Set up
$$\begin{array}{ccccccc} R^n & \xrightarrow{M} & R^n & \longrightarrow & V & \longrightarrow & 0 \\ \downarrow Q & & \downarrow P & & \downarrow C & & \\ R^n & \longrightarrow & R^n & \longrightarrow & R^n & \longrightarrow & 0 \end{array}$$

3. * Find M

4. * Find P, Q from C
   * Find C from P, Q

5. $\begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix} \sim \begin{pmatrix} 1 & 0 \\ 0 & ab \end{pmatrix}$ & the s&t of $x-\lambda_n$, $D_{n-1} = \prod_{i=1}^{n-1}(x-\lambda_i)$

6. Something about $(x-\lambda)^s \leftrightarrow \begin{pmatrix} \lambda & & 0 \\ 1 & \lambda & \\ 0 & 1 & \lambda \end{pmatrix}$

Discuss the Final.

HW. HW5 due on Thursday! [but can be handed in today]

Analysis Killer: Hilbert B, tomorrow 2-3 @ SS 1070.

The Best HW Ever: Open All boxes. (We'll only set the language)

Cayley-Hamilton. Let R be any commutative ring, let $A \in M_{n\times n}(R)$, let $\chi_A(t) = \det(tI - A) \in R[t]$. Then $\chi_A(A) = 0$.

Proof II. Recall that every matrix B has an "adjoint" B^* s.t. $B^*B = BB^* = \det(B)\cdot I$. Then

$$\underbrace{(tI-A)^*}_{\sum B_k t^k}(tI-A) = \chi_A(t)I$$

as elements of $M_n R[t]$ & even $C_A[t]$, where $C_A = \{B : AB = BA\}$

There is a well-defined $ev_A : C_A[t] \to C_A[t]$. Applying to both sides, get
&multiplicative

$$0 = \left(\sum B_k A^k\right)\cdot(A - A) = \chi_A(A)I \qquad \square$$
