- Course evaluations today!
- Linear equations and matrices.
- Go as far as theory will go...
- ... and then switch to practice.
- Maybe start with dets.

board · q

$\begin{cases} 2x - 7y = -3 \\ 2y - x = 0 \end{cases}$

$(x,y) = (2,1)$

In this case,

$A = \begin{pmatrix} 2 & -7 \\ -1 & 2 \end{pmatrix}$ $A^{-1} = \begin{pmatrix} -\frac{2}{3} & -\frac{7}{3} \\ -\frac{1}{3} & -\frac{2}{3} \end{pmatrix}$

In general

$a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n = b_1$

$\vdots$

$a_{m1}x_1 + \ldots + a_{mn}x_n = b_m$

$\Rightarrow$

$Ax = b$ where $A = \begin{pmatrix} a_{11} & \cdots & a_{1n} \\ a_{m1} & \cdots & a_{mn} \end{pmatrix}$

$b = \begin{pmatrix} b_1 \\ b_m \end{pmatrix}$

"solving for the coordinates of an unknown vector"

Taxonomy: $Ax = 0$: homogeneous system of lin. eqns

$Ax = b$: inhomogeneous system of lin eqns

If we are lucky and A is invertible, then $x = A^{-1}b$. Often we are, but often we are not.

The homogeneous case 1. x is a soln iff $x \in \ker A$.
(so the set of soln is always a subspace of F^n)

2. 0 is always a soln

The general case

1. A soln exists iff $b \in R(A) = \text{im}(A) = \text{col-spc}(A)$.

2. If x_0 is a soln then x_1 is also a soln iff

$x_1 = x_0 + x$ where x is a soln of the homogeneous

eqn $Ax = 0$ ("affine subspace")

$Ax = b \iff \underbrace{EAx}_{A'x} = Eb$ $(A \mid b) \xrightarrow[\text{ops}]{\text{row}} (C \mid d)$
$\qquad\qquad\qquad A'x = b'$

If C is r.r.e.f.:

$\begin{bmatrix} 1 & 0 & 2 & 9 & 0 & e \mid d_1 \end{bmatrix}$ non-zero pivotal

Example:

$$\left[\begin{array}{cccccc|c} 1 & 0 & 2 & 9 & 0 & e & d_1 \\ 0 & 1 & -3 & 7 & 0 & \pi & d_2 \\ 0 & 0 & 0 & 0 & 1 & 2 & d_3 \\ 0 & 0 & 0 & 0 & 0 & 0 & d_4 \end{array}\right]$$

} non-zero $\quad$ Pivotal
$\quad$ rows $\quad\quad$ rows

} zero rows / non-pivotal
$\quad\quad\quad\quad\quad$ rows.

pivotal col's

non-pivotal col's

1. Sol'n exist iff the d_i's in the non-pivotal rows are 0.

2. The x_j's corresponding to the non-pivotal col's can be set arbitrarily, the x_j's corresponding to the pivotal rows are then fixed.

Example

$$\begin{aligned} 2x_1 + 3x_2 + x_3 + 4x_4 - 9x_5 &= 17 \\ x_1 + x_2 + x_3 + x_4 - 3x_5 &= 6 \\ x_1 + x_2 + x_3 + 2x_4 - 5x_5 &= 8 \\ 2x_1 + 2x_2 + 2x_3 + 3x_4 - 8x_5 &= 14, \end{aligned}$$

$$\begin{pmatrix} 2 & 3 & 1 & 4 & -9 & 17 \\ 1 & 1 & 1 & 1 & -3 & 6 \\ 1 & 1 & 1 & 2 & -5 & 8 \\ 2 & 2 & 2 & 3 & -8 & 14 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & 1 & 1 & 1 & -3 & 6 \\ 2 & 3 & 1 & 4 & -9 & 17 \\ 1 & 1 & 1 & 2 & -5 & 8 \\ 2 & 2 & 2 & 3 & -8 & 14 \end{pmatrix} \longrightarrow$$

and so...

$$\begin{pmatrix} 1 & 1 & 1 & 1 & -3 & 6 \\ 0 & 1 & -1 & 2 & -3 & 5 \\ 0 & 0 & 0 & 1 & -2 & 2 \\ 0 & 0 & 0 & 1 & -2 & 2 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & 1 & 1 & 1 & -3 & 6 \\ 0 & 1 & -1 & 2 & -3 & 5 \\ 0 & 0 & 0 & 1 & -2 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \longrightarrow$$

$$\begin{pmatrix} 1 & 1 & 1 & 0 & -1 & 4 \\ 0 & 1 & -1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & -2 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & 0 & 2 & 0 & -2 & 3 \\ 0 & 1 & -1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & -2 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} = \begin{pmatrix} -2t_1 + 2t_2 + 3 \\ t_1 - t_2 + 1 \\ t_1 \\ 2t_2 + 2 \\ t_2 \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \\ 0 \\ 2 \\ 0 \end{pmatrix} + t_1 \begin{pmatrix} -2 \\ 1 \\ 1 \\ 0 \\ 0 \end{pmatrix} + t_2 \begin{pmatrix} 2 \\ -1 \\ 0 \\ 2 \\ 1 \end{pmatrix}$$