240Algebra I-091006, Hours 11-12

October-01-09
2:13 PM

TA OH reminder!

TA Consultation Hours: Monday 6PM-8PM and Thursdays 11AM-1PM at Bahen 6283.

Goal: Every V.S. has a basis.

on board

Def: 1. $\operatorname{Span} S := \{\sum a_i u_i : a_i \in F, u_i \in S\}$

2. $S \subset V$ is "lin indep" if $\sum a_i u_i = 0$ for distinct $u_i \in S \Rightarrow \forall i\ a_i = 0$.

<u>Claim</u> If S is lin indep in V and $v \in V \setminus S$, then $S \cup \{v\}$ is lin. dep. iff $v \in \operatorname{Span}(S)$.

"wasteful"

<u>Def</u> Basis $\beta \subset V$

<u>Examples</u>: 1. $\emptyset$ for $\{0\}$.

2. e_i for F^n 3. E^{ij} for $M_{m \times n}(F)$

4. $(1, x, \dots x^n)$ for $P_n(F)$

5. $(1, x, \dots\quad)$ for $P(F)$

6. $\left\{\begin{pmatrix}1\\1\end{pmatrix}, \begin{pmatrix}1\\-1\end{pmatrix}\right\}$ for $\mathbb{R}^2$.

<u>Thm</u> A subset $\beta \subset V$ is a basis iff every $v \in V$ can be expressed in a unique way as a l.c. of elements of β.

<u>Thm</u> If a finite set S generates a V.S. V, then there is a subset $\beta \subseteq S$ which is a basis of V

<u>Pf</u> Let β be a lin indep subset of S which is of maximal size. The every $v \in S \setminus \beta$ satisfies $v \in \operatorname{span} \beta$, so $S \subset \operatorname{span} \beta$, so $\operatorname{span} S \subset \operatorname{span} \beta$.

Our first non-language theorem:

<u>Thm</u> If a v.s. V has a finite basis, then every other basis of V has the same number of elements in it.

<u>Def</u> If V has a finite basis, we say that

it is "finite-dimensional" and let

$$\dim V := \begin{pmatrix} \text{The number of elements} \\ \text{in (any) basis of } V \end{pmatrix}$$

<u>Lemma</u> (the replacement lemma)

$|G| = n$, $\operatorname{span} G = V$, L lin indep

$\Longrightarrow |L| \leq n$ & $\exists H \subset G$ with

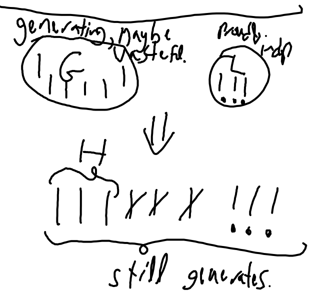
$|H| = n - |L|$ and $\operatorname{span}(H \cup L) = V$

Examples as before:

$\{0\}$, F^n,

$M_{m \times n}$, $P_n(F)$,

$P(F)$



Pf of theorem from lemma.