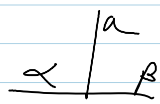
AKT-091006, Hour 12: Low algebra, the gl(N) example

October-05-09
6:09 PM

1. Review Lie algs.
2. The low algebra statement:

$$W_{\mathfrak{g},R} : \quad \begin{matrix} f_{abc} \\ f^{ab} \end{matrix} \qquad R(X_a)\cdot e_\alpha = r^\beta_{a\alpha} e_\beta$$



3. gl_N

locator map:

	u	v	w
Top			
comb			
Low			
High			

Exercise Let $D \in \mathcal{A}(\emptyset)$. Prove that

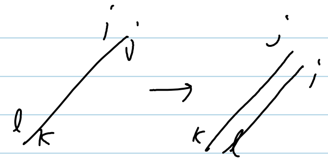
$$W_{gl(2)}(D) = 0 \implies W^{top}_{gl(N)} = 0.$$

The gl_N calculation $\quad a \leftrightarrow (ij)$

$$X_{ij} = i\begin{pmatrix} & j & \\ & & \end{pmatrix} \qquad X_{ij}X_{kl} = \delta_{jk}X_{il}$$

$$[X_{ij}, X_{kl}] = \delta_{jk}X_{il} - \delta_{il}X_{kj}$$

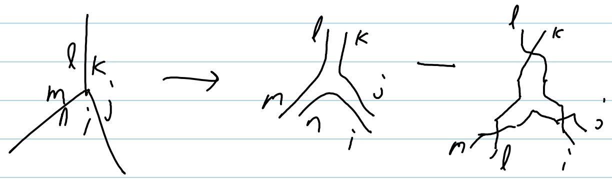
$$t_{(ij)(kl)} = tr\, X_{ij}X_{kl} = \delta_{jk}\delta_{il}$$



$t^{(ij)(kl)} =$ same; indeed

$$t_{ij,kl}\, t^{kl,mn} = \sum_{k,l} \delta_{jk}\delta_{il}\delta^{kn}\delta^{lm} = \delta_{jn}\delta_{im}$$

$$F_{ij,kl,mn} = \langle [X_{ij}, X_{kl}], X_{mn}\rangle = \langle \delta_{jk}X_{il} - \delta_{il}X_{kj}, X_{mn}\rangle$$
$$= \delta_{jk}\delta_{lm}\delta_{in} - \delta_{il}\delta_{jm}\delta_{kn}$$



$$r^l_{(ij)k} = \delta_{jk}\delta^l_i$$