AKT-090917, Hours 4-5: Definition of finite type, Jones and the weight system of Jones, some dimensions

September-16-09
4:44 PM

See "Some Computations" at

http://katlas.math.toronto.edu/drorbn/index.php?title=06-1350/Class_Notes_for_Thursday_October_12

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0. A few further web words, Nadish's work.

Embarassing:

$$\langle\emptyset\rangle = 1; \qquad \langle \bigcirc L\rangle = (-A^2 - B^2)\langle L\rangle; \qquad \langle \times \rangle = A\langle \asymp \rangle + B\langle)(\rangle;$$

$$J(L) = (-A^3)^{w(L)} \frac{\langle L\rangle}{\langle \bigcirc \rangle}\Bigg|_{A\to q^{1/4}}, \to \quad A^2 \to -q^{-1/2} \quad A \to i q^{-1/4} \qquad A^4 \to 1/q \quad -A^2 - A^{-2} \to q^{1/2} + q^{-1/2}$$

Much of what we'll do is in "On the Vassiliev Knot invariants, DBN 1992,"

http://www.math.toronto.edu/~drorbn/papers/OnVassiliev/index.html

1. Define "Finite type invariant", $\mathcal{D}$, $W_f: \mathcal{D} \to A$
2. Thm $J(K)(e^x) = \sum V_n(K) x^n$; J_n is of type n.

$$\nearrow\!\!\!\!\nwarrow \;\to\; A \asymp + A^{-1})(\;\to\; -A^4 \asymp - A^2)(= -q^{-1} \asymp + q^{-\frac12})($$

$$\nwarrow\!\!\!\!\nearrow \;\to\; A^{-1} \asymp + A)(\;\to\; -A^{-4} \asymp - A^{-2})(= -q \asymp + q^{+\frac12})($$

$$q\, J(\nearrow\!\!\!\!\nwarrow) - q^{-1} J(\nwarrow\!\!\!\!\nearrow) = (q^{1/2} - q^{-\frac12}) J(\uparrow\uparrow)$$

"The skein relation" $\qquad J(\bigcirc^k) = (q^{1/2} + q^{-\frac12})^{k-1}$

Aside This & $J(\bigcirc)$ define J, and likewise

$$A(\nearrow\!\!\!\!\nwarrow) - A(\nwarrow\!\!\!\!\nearrow) = z A(\uparrow\uparrow)$$

& $q^{+N/2} H(\nearrow\!\!\!\!\nwarrow) - q^{-N/2} H(\nwarrow\!\!\!\!\nearrow) = (q^{1/2} - q^{-1/2}) H(\uparrow\uparrow)$ $\Big\}$ define Alex, HOMFLY-PT

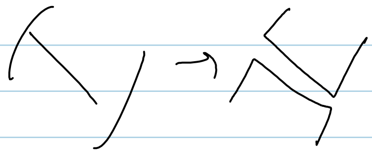
$$H(\bigcirc^k) = \frac{q^{+N/2} - q^{-N/2}}{q^{1/2} - q^{-1/2}}$$

⋮

$$H(O^k) = \frac{q^{+N/2} - q^{-N/2}}{q^{1/2} - q^{-1/2}}$$

3. Derive W_{J_n}, W_{H_n}, W_{A_n}

W_{A_n}:



4 FI, 4T.

5.

m	0	1	2	3	4	5	6	7	8	9	10	11	12
$\dim \mathcal{A}^r_m$	1	0	1	1	3	4	9	14	27	44	80	132	232
$\dim \mathcal{A}_m$	1	1	2	3	6	10	19	33	60	104	184	316	548
$\dim \mathcal{P}_m$	0	1	1	1	2	3	5	8	12	18	27	39	55